DEPARTMENT OF CIVIL ENGINEERING: IIT DELHI

II SEMESTER 2013-2014 **DESIGN OF INDUSTRIAL STRUCTURES** **CEL 432**

MINOR II **Max. Marks: 20** **Time Limit: 1 hour**

Course Coordinator: Dr. Alok Madan

1. Based on theory of shells, obtain the complete solutions of the radial displacement 'w', meridional moment 'M_x', circumferential moment 'M_θ', radial shear force Q_x and tangential (circumferential) normal force 'N_θ' for a circular **cylindrical pressure vessel** fabricated as a **thin cylindrical steel shell** of mean radius 'a', height 'L' and shell thickness 't' with **fixed thick liners (rigid diaphragms) at both ends (fixed end conditions).** The pressure vessel contains a gas with **a uniform internal pressure p_0** and is installed with its longitudinal axis aligned vertically. **Consider the self-weight of the shell (unit weight γ_s) and uniform internal pressure to derive the complete analytical solutions in a closed form (in terms of x, a, t, L, E, λ, ν, γ_s and p_0).** Some useful formulae are given below. (15 marks)

2. For an uniform internal pressure p_0 = 45 kN/m^2, a mean radius a = 1 m, shell thickness t = 25 mm and height L = 4 m, (i) determine, using the analytical solution obtained above, the numerical values of the meridional moment 'M_x', and circumferential normal force 'N_θ' at the **lower end of the shell.** (ii) Calculate the numerical value of 'M_x' at **mid-height of the shell** and express this value as a fraction of 'M_x' at the lower end of the shell. Assume unit weight of steel γ_s = 77 kN/m^3, Poisson's ratio υ = 0.15 and elastic modulus E_s = 2x10^5 MPa. (5 marks)

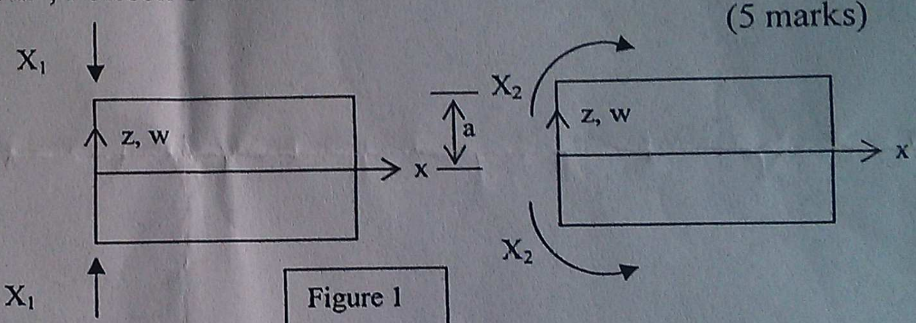

Figure 1

SOME USEFUL FORMULAE:

Bending Solution in a cylindrical shell due to edge force X_1 and edge moment X_2 (Fig. 1)

$$M_x = \frac{-X_1 a}{\lambda}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}\right) \qquad M_x = \sqrt{2}X_2\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}-\frac{\pi}{4}\right)$$

$$Q_x = -\sqrt{2}X_1\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}+\frac{\pi}{4}\right) \qquad Q_x = -\frac{\sqrt{2}X_2\lambda}{a}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}\right)$$

$$N_\theta = -2X_1\lambda\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}\right) \qquad N_\theta = \frac{-2\sqrt{2}X_2\lambda^2}{a}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}-\frac{\pi}{4}\right)$$

$$M_\theta = \nu M_x \qquad M_\theta = \nu M_x$$

$$w = -\frac{X_1 a^3}{2K\lambda^3}\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}\right) \qquad w = \frac{-X_2 a^2}{\sqrt{2}K\lambda^2}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}-\frac{\pi}{4}\right)$$

$$\frac{dw}{dx} = \frac{X_1 a^2}{\sqrt{2}K\lambda^2}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}+\frac{\pi}{4}\right) \qquad \frac{dw}{dx} = \frac{X_2 a}{K\lambda}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}-\frac{\pi}{2}\right)$$

where, $K = \dfrac{Et^3}{12(1-\nu^2)}$; $\lambda^4 = 3(1-\nu^2)\dfrac{a^2}{t^2}$