

Department of Applied Mechanics
Indian Institute of Technology Delhi
Major, First Semester 2021-22

Course Title: Advanced Finite Element Method

Course No.: AML 805

Date: 18 November 2021

Duration: 2.5 Hrs (9.00-11.30 AM)

Maximum Marks: 150

Instructions:

- (i) You can refer your notes and books. It is an open notes, open book examination.
- (ii) Answer all the questions. Marks are indicated against each question. There are total 9 Questions.
- (iii) Write answers in your own handwriting.
- (iv) Start answer of each question on a new page.
- (v) Stop writing at 11.30 AM and start scanning and uploading pdf in gradescope. You should finish uploading answers before 11.50 AM.
- (vi) After uploading pdf, select pages for each question.
- (vii) If you have any doubt, call me on my mobile: 9213227603.

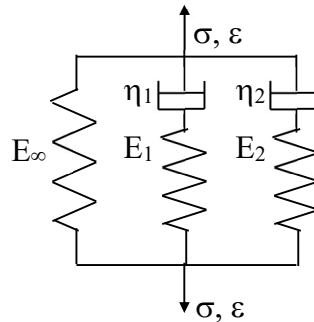
Q.1: The weak form of equilibrium equations for a 2D plane stress problem is given by:

$$\int_{V^0} \delta \{\mathbf{E}\}^T \{\mathbf{S}\} dV^0 - \int_{V^0} \delta \{\mathbf{U}\}^T \{\mathbf{f}_b\} dV^0 = 0 \text{ where}$$

$$\{\mathbf{E}\} = \begin{Bmatrix} E_{XX} \\ E_{YY} \\ E_{XY} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial U}{\partial X} + \frac{1}{2} \left(\frac{\partial V}{\partial X} \right)^2 \\ \frac{\partial V}{\partial Y} + \frac{1}{2} \left(\frac{\partial V}{\partial Y} \right)^2 \\ \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} + \frac{\partial V}{\partial X} \frac{\partial V}{\partial Y} \end{Bmatrix}; \{\mathbf{S}\} = \begin{Bmatrix} S_{XX} \\ S_{YY} \\ S_{XY} \end{Bmatrix}; \{\mathbf{U}\} = \begin{Bmatrix} U \\ V \end{Bmatrix}; \{\mathbf{f}_b\} = \begin{Bmatrix} f_{bX} \\ f_{bY} \end{Bmatrix}; \{\Delta \mathbf{S}\} = [\mathbf{C}_T] \{\Delta \mathbf{E}\}; U$$

and V are displacements along X and Y directions. Derive the incremental element level governing equation considering 4-noded isoparametric element. [20]

Q. 2: For the viscoelastic model shown, derive the relation: $E(t) = E_\infty + \sum_{i=1}^2 E_i e^{-\frac{t}{\tau_i}}$. [10]



Q. 3: State of stress at a point in a body is given by: $\sigma_{xx} = 120$ MPa, $\sigma_{yy} = -80$ MPa, $\tau_{xy} = 0$ MPa, $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$. Consider isotropic strain hardening, von Mises yield function and associative flow rule ($E = 200$ GPa, $\nu = 0$, plastic modulus (H) = 200 MPa, initial yield stress (σ_y) = 200 MPa).

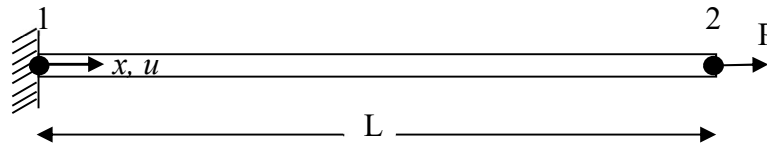
(i) Is the current state of stress elastic or elasto-plastic? [3]

(ii) For the next load step, finite element solution gives incremental strains: $\Delta\varepsilon_{xx} = 0.0009$, $\Delta\varepsilon_{yy} = 0.0009$, $\Delta\gamma_{xy} = \Delta\gamma_{zz} = \Delta\gamma_{xz} = \Delta\gamma_{yz} = 0$. Show that the new state of stress calculated assuming elastic behavior lies outside the yield surface. [5]

(iii) Calculate the factor β such that state of stress at the point reaches to the yield surface for strain increments: $\Delta\varepsilon_{xx} = 0.0009 \beta$, $\Delta\varepsilon_{yy} = 0.0009 \beta$, $\Delta\gamma_{xy} = \Delta\gamma_{zz} = \Delta\gamma_{xz} = \Delta\gamma_{yz} = 0$. [12]

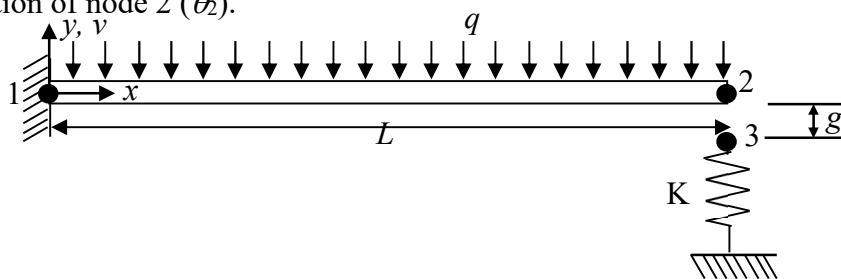
Q. 4: Derive the expressions for first (I_1) and second (I_2) invariants of Right Cauchy Green deformation tensor in terms of Green Lagrange Strain tensor components. For incompressible material ($I_3=1$) with $E_{YY}=E_{ZZ}$, express E_{YY} and E_{ZZ} in terms of E_{XX} if $E_{XY} = E_{YZ} = E_{ZX} = 0$. [10]

Q. 5: Perform one iteration for the solution of the truss member as shown ($EA = 100 \text{ N}$, $L = 1 \text{ m}$, $P = 10 \text{ N}$, $q = 0$). Model the truss using one two-noded truss element considering Finite Deformation. The initial guess is given by $\{u_1 \ u_2\}^T = \{0.0 \ 0.1\}^T$. [20]

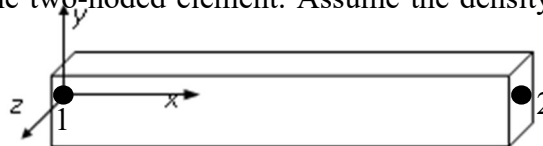


You may note that due to boundary conditions: $\Delta u_1 = 0$.

Q. 6: For the cantilever beam as shown ($EI = 1 \text{ N m}^2$, $L = 1 \text{ m}$, $q = 0.16 \text{ N/m}$, $K = 12 \text{ N/m}$, $g = 0.01 \text{ m}$). Model the beam using one Euler-Bernoulli beam element and find displacements of nodes 2 (v_2) and 3 (v_3) and rotation of node 2 (θ_2). [20]



Q. 7: Derive the mass matrix of a two-noded frame element with the displacements: $u(x, y, t) = u_0(x, t) - y\theta_z(x, t)$; $v(x, y, t) = v(x, t)$. θ_z is an independent variable. Interpolate u_0 , v and θ_z using shape functions for the two-noded element. Assume the density and cross-section constant within the element. [20]



Q. 8: For a 4-noded isoparametric Mindlin plate bending reduced integration element with nodal coordinates as: $(x_1, y_1) = (0, 0)$; $(x_2, y_2) = (1, 0)$; $(x_3, y_3) = (1, 1)$; $(x_4, y_4) = (0, 1)$, find the hourglass stabilization matrix corresponding to transverse displacement w . You can leave your answer in terms of stabilization constant c_1 . [10]

Q. 9: Set-up the element level governing equation for the solution of nonlinear differential equation:

$-\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2 - 4 = 0$. Use two-noded element for the interpolation of u and take element length as

l . In the weak form, do the integration by parts only for the first term involving second derivative of u .

[20]