

**Department of Applied Mechanics**  
**Indian Institute of Technology Delhi**  
**Major, First Semester 2020-21**

**Course Title: Advanced Finite Element Method**

**Course No.: AML 805**

**Date: 11 January 2020**

**Duration: 2.5 Hrs (9.00-11.30 AM)**

**Maximum Marks: 130**

**Instructions:**

- (i) You can refer your notes and books. It is an open notes and open book examination.
- (ii) Answer all questions. Marks are indicated against each question. There are total 9 Questions.
- (iii) Write answers in your own handwriting.
- (iv) Start answer of each question on a new page.
- (v) Stop writing at 11.30 AM and start scanning and uploading pdf in gradescope. You should finish uploading answers before 11.50 PM.
- (vi) After uploading pdf, select pages for each question.
- (vii) If you have any doubt, call me on my mobile: 9213227603.

**Q. 1:** Consider a square of side 2 units with initial location of its centre at  $\{X \ Y \ Z\}^T = \{0 \ 0 \ 0\}^T$ . The coordinates  $\{x \ y \ z\}^T$  of the deformed configuration of the square are given by:

$$x = 4 + X + 1.5 Y; \quad y = 3 + 1.5 Y; \quad z = Z.$$

If the components of Cauchy Stress tensor are:  $\sigma_{xx} = 100$  MPa,  $\sigma_{yy} = 200$  MPa,  $\sigma_{zz} = -50$  MPa,  $\tau_{xy} = 100$  MPa,  $\tau_{yz} = \tau_{zx} = 0$ ; Find the components of First Piola-Kirchhoff Stress and Second Piola-Kirchhoff Stress tensors. [10]

**Q. 2:** The Green-Lagrange strain in a truss member oriented in 2D space is given by:

$$\epsilon_{xx} = \frac{dU}{dX} + \frac{1}{2} \left( \left( \frac{dU}{dX} \right)^2 + \left( \frac{dV}{dX} \right)^2 \right) \text{ where components of the displacement vector } \{U \ V\}^T \text{ with}$$

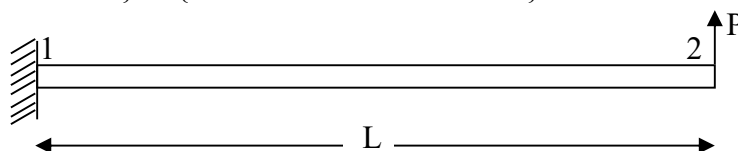
respect to initial un-deformed configuration are function of X coordinate only. The strain energy ( $\pi$ )

stored in the member is given by:  $\pi = \frac{1}{2} \int_0^L EA (\epsilon_{xx})^2 dX$ . For a two-noded element, give the following:

- (i) Strain-displacement matrices:  $\mathbf{B}$  and  $\mathbf{B}_{NL}$ , (ii) Final expression for tangent stiffness matrix:  $\mathbf{K}_T^e$ ,
- (iii) Right hand side of element level incremental equation:  $\mathbf{F}^e$ . [20]

**Q. 3:** Write the steps to perform one iteration for the solution of the cantilever beam as shown ( $EA = 100$  N,  $EI = 1$  N m<sup>2</sup>,  $L = 1$  m,  $P = 0.3$  N). Model the beam using one Euler-Bernoulli beam element considering geometric nonlinearity. The starting solution is given by

$$\{U_{01} \ W_1 \ \theta_1 \ U_{02} \ W_2 \ \theta_2\}^T = \{0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.1 \ 0.15\}^T. \quad [20]$$



**Q. 4:** The total potential energy for a beam is given by:

$$I = \frac{1}{2} \int_0^L \left[ a \left( \frac{du_0}{dx} \right)^2 + b \left( \frac{d^2v}{dx^2} \right)^2 + c \left( \frac{d\theta}{dx} \right)^2 + 2e \frac{d^2v}{dx^2} \frac{d\theta}{dx} + f \left( \frac{dv}{dx} + \theta \right)^2 \right] dx.$$

For a two noded beam element, suggest the approximate solutions for  $u_0$ ,  $v$ ,  $\theta$ , and state the degrees of freedom at each node. How are the continuity and **inter-element compatibility** conditions satisfied by your assumed solution? You need not derive the shape functions. [10]

**Q. 5:** Equation of motion for a system is given below:

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_1 \end{Bmatrix} + \begin{bmatrix} 20 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} t \\ 0.5t \end{Bmatrix}$$

(i) Find the undamped natural frequencies of the system. [5]

(ii) For the system, estimate  $\Delta t$  for numerically stable central difference scheme. [2]

(iii) Find displacement vector at  $t = 0.1$  second using Newmark's time integration scheme taking  $\Delta t = 0.1$  seconds and zero initial conditions. [8]

**Q. 6:** The differential equation governing the axial deformation ( $u$ ) of an elastic rod of length  $L$  subjected to distributed load  $q$  and temperature change  $T$  is given by:

$$\frac{d}{dx} \left[ EA \left( \frac{du}{dx} - \alpha T \right) \right] + q = 0, \quad \text{subject to the boundary conditions: } u = 0 \text{ at } x = 0 \text{ and } L.$$

$E$ ,  $A$ ,  $\alpha$ ,  $T$  and  $q$  are known and  $u$  is an unknown. Derive the associated Integral Functional. [10]

**Q. 7:** For a three-noded truss element, the potential energy ( $V$ ), kinetic energy ( $T$ ) and virtual work of non-conservative forces ( $\delta W$ ) are given by:

$$V = \frac{1}{2} \{\mathbf{d}^e\}^T [\mathbf{K}^e] \{\mathbf{d}^e\}; \quad T = \frac{1}{2} \{\dot{\mathbf{d}}^e\}^T [\mathbf{M}^e] \{\dot{\mathbf{d}}^e\}; \quad \delta W = -\delta \{\mathbf{d}^e\}^T [\mathbf{C}^e] \{\dot{\mathbf{d}}^e\} + \delta \{\mathbf{d}^e\}^T \{F^e\}.$$

Derive the equation of motion using Hamilton's principle. [15]

**Q.8:** In the associated flow rule, incremental plastic strains are given by  $\{d\varepsilon^p\} = \lambda \left\{ \frac{\partial F}{\partial \sigma} \right\}$ . Taking von

Mises yield function with isotropic strain hardening, show that  $\lambda = d\varepsilon_{eq}^p$  where

$$d\varepsilon_{eq}^p = \sqrt{\frac{2}{3} \left( (d\varepsilon_{xx}^p)^2 + (d\varepsilon_{yy}^p)^2 + (d\varepsilon_{zz}^p)^2 + ((d\gamma_{xy}^p)^2 + (d\gamma_{yz}^p)^2 + (d\gamma_{zx}^p)^2) / 3 \right)}. \quad [15]$$

**Q. 9:** Consider two elements that are in potential contact as shown. The nodal coordinates are:

Node	x	y	Node	x	y
1	0	0	5	0.5	1.5
2	1.5	0	6	1.8	2.3
3	1.5	1	7	1.3	3.1
4	0	1	8	0	2.4

For a point located at the center of contactor edge 5-6, determine its contact point on the target line. Also compute the gap distance. [15]

