

Department of Applied Mechanics
Indian Institute of Technology Delhi
Minor, First Semester 2020-21

Course Title: Advanced Finite Element Method

Course No.: AML 805

Date: 11 November 2020

Duration: 1.5 Hrs (1.00-2.30 PM)

Maximum Marks: 90

Instructions:

- (i) You can refer your notes and books. It is an open notes, open book examination.
- (ii) Answer all the questions. Marks are indicated against each question. There are total 7 Questions.
- (iii) Write answers in your own handwriting.
- (iv) Start answer of each question on a new page.
- (v) Stop writing at 2.30 PM and start scanning and uploading pdf in gradescope. You should finish uploading answers before 2.50 PM.
- (vi) After uploading pdf, select pages for each question.
- (vii) If you have any doubt, call me on my mobile: 9213227603.

Q. 1 The coupled differential equations governing transverse deflection (v) and slope (θ) of a beam are as follows:

$$-\frac{d}{dx} \left(GA \left(\frac{dv}{dx} - \theta \right) \right) - q = 0 \quad (\text{Equilibrium of Forces in the transverse direction})$$

$$-\frac{d}{dx} \left(EI \frac{d\theta}{dx} \right) - GA \left(\frac{dv}{dx} - \theta \right) = 0 \quad (\text{Equilibrium of Moments})$$

where G, E, A, I, q are known constants. Length of the beam is L .

- (i) Obtain the weak form of the problem, [7]
- (ii) Identify Primary and Secondary variables, [2]
- (iii) Obtain the associated Integral Functional. [6]

Hint: Weighted residual statement of the problem can be written by multiplying the first equation with δv , second with $\delta \theta$ and integrating the sum of the resulting expressions from $x = 0$ to $x = L$.

Q. 2: The weak form of the differential equation governing the buckling of a beam under axial compressive force is given by:

$$w \frac{d}{dx} \left(EI \frac{d^2 v}{dx^2} \right) \Big|_0^L - \frac{dw}{dx} EI \frac{d^2 v}{dx^2} \Big|_0^L + \int_0^L \frac{d^2 w}{dx^2} EI \frac{d^2 v}{dx^2} dx + w P \frac{dv}{dx} \Big|_0^L - \int_0^L \frac{dw}{dx} P \frac{dv}{dx} dx = 0 \quad \text{where } EI, P \text{ are}$$

constants, v is an unknown function of x and w is weight function. Identify primary and secondary variables. [5]

Q. 3: Consider a square of side 2 units with initial location of its centre at $\{X \ Y \ Z\}^T = \{0 \ 0 \ 0\}^T$. The coordinates $\{x \ y \ z\}^T$ of the deformed configuration of the square are given by:

$$x = 4 + X + 1.5 Y; \quad y = 3 + 1.5 Y; \quad z = Z.$$

- (i) Compute the components of the deformation gradient tensor and write it in matrix form. [3]
- (ii) Compute the components of the Green-Lagrange and Eulerian Almansi strain tensors. [7]

Q. 4: Suggest the approximate solutions (for $u_0, v_0, w_0, \theta_x, \theta_y$) for a 4-noded conforming rectangular plate element based on a higher order plate theory with through the thickness approximation of displacements as:

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y) - \frac{4}{3h^2} z^3 \left(\theta_x + \frac{\partial w_0}{\partial x} \right), \quad v(x, y, z) = v_0(x, y) + z\theta_y(x, y) - \frac{4}{3h^2} z^3 \left(\theta_y + \frac{\partial w_0}{\partial y} \right)$$

$$w(x, y, z) = w_0(x, y)$$

You may note that θ_x, θ_y are independent of w_0 and $\frac{\partial w_0}{\partial x}, \frac{\partial w_0}{\partial y}$ are appearing in the displacement

approximation above. You need not derive the shape functions. Specify degrees of freedom at each node. [15]

Q. 5: The strain displacement relations for a curved beam element are given as:

$$\varepsilon_{xx} = \frac{du_0}{dx} + w/R + z \frac{d\theta}{dx}; \quad \gamma_{xz} = \frac{dw}{dx} + \theta \quad \text{where } R \text{ (known constant) is radius of curvature and } u_0, w,$$

θ are functions of x . Thin and shallow beam elements depict membrane and shear locking.

Membrane locking is due to membrane strain: $\frac{du_0}{dx} + w/R$ approaching to zero when beam becomes

thin and shallow. Suggest different methods to eliminate membrane locking. You need to suggest suitable interpolation for u_0 and w to interpolate the membrane strain. [10]

Q. 6: Stress in a viscoelastic rod under uniaxial state of stress is given by: $\sigma_{xx} = \int_0^t E(t-t') \frac{d\varepsilon_{xx}}{dt'} dt'$

where $E(t) = E_\infty + \sum_{i=1}^N E_i e^{-\frac{t}{\tau_i}}$. Derive the relation between incremental stress $\Delta\sigma_{xx}$ (=

$$\sigma_{xx}(t + \Delta t) - \sigma_{xx}(t)) \text{ and incremental strain } \Delta\varepsilon_{xx} (= \varepsilon_{xx}(t + \Delta t) - \varepsilon_{xx}(t)). \quad [15]$$

Q. 7: Stresses at a point in a body under plane stress condition is given by: $\sigma_{xx} = 250$ MPa, $\sigma_{yy} = 150$ MPa, $\tau_{xy} = 100$ MPa, $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$. Obtain tangent constitutive matrix considering isotropic strain hardening, von Mises yield function and associative flow rule. The material properties are: $E = 200$ GPa, $\nu = 0$, plastic modulus (H) = 200 MPa, initial yield stress (σ_{Y0}) = 200 MPa. [20]