(8)

APPLIED MECHANICS DEPARTMENT

AML110: MECHANICS

SEMESTER – II, 2006-2007 MAJOR TEST TIME: 8 – 10 AM

MAX. MARKS: 100

Note: Answer all questions.

1. The mass and axial moment of inertia of the front and rear wheel assembly (Fig.Q1) are m and I_0. There is no slip at all contacts. The springs are undeformed for $x_1 = 0, x_2 = 0$. x_2 is the displacement of the centre of the disc w.r.t. ground.
   (a) Express the virtual work done by P_1, P_2, and C using the generalized co-ordinates x_1 and x_2, and find the generalized forces Q_1^{nc} and Q_2^{nc}.
   (b) Write the expressions of T and V for the two-dof system.
   (c) Derive the Lagrange's equation of motion for the system. (4,3,4)

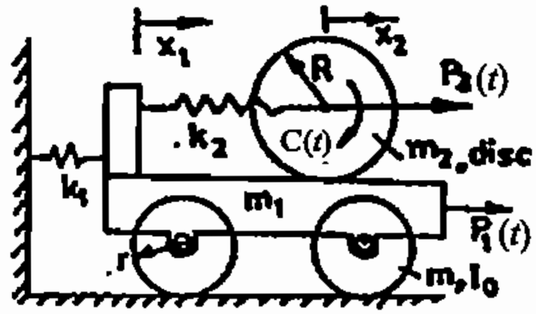

Fig. Q1

2. A light rod AB is pinned to a block of weight W at A. Also at A are two identical springs k (Fig. Q2). The potential energy for the two springs is given by $V(\theta) = kL^2 \sin^2 \theta$. (a) Determine the positions of equilibrium. (b) Determine the condition of stability in respect of W for the vertical position $(\theta = 0)$. (7)

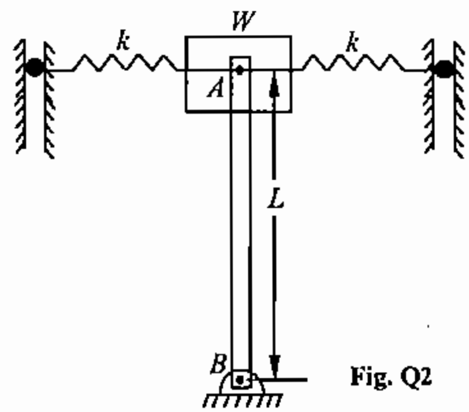

Fig. Q2

3. Determine the forces in members BC and FG of the truss shown in Fig. Q3. Use method of section. (5)

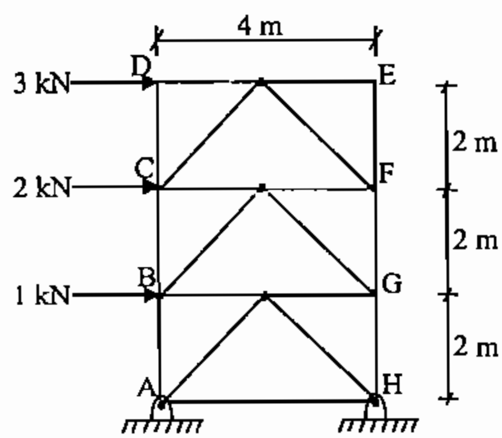

Fig. Q3

4. The governing equation for the trajectory of the mass centre of a body subject to central force of the form $\underline{F} = F_r(r)\underline{e}_r$ is given by

$$\frac{d^2u}{d\phi^2} + u = -F_r(u)/mh_0^2u^2 \text{ with } u = 1/r.$$

Obtain the trajectory of the mass centre of a body in central force motion under gravitational force using the above equation. (5)

5. Draw the SF and BM diagrams labeling all key points for the beam shown (Fig. Q5). (12)

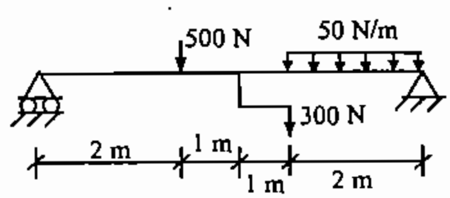

Fig. Q5

6. A 60 N bar AB is pinned at A and rests on a 180 N cylinder with centre of mass C (Fig. Q6). The diameter of the cylinder is 0.3 m and the coefficient of static friction is 0.35 between all surfaces in contact. Determine the largest force P for which equilibrium is maintained. (10)

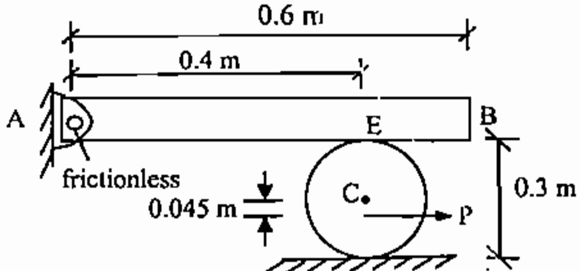

Fig. Q6

7. The crank OA of the mechanism shown (Fig. Q7) rotates at constant angular velocity ω. The crank has a telescopic arm whose length $r(t)$ is controlled by a hydraulic cylinder at a constant rate $\dot{r} = v$. Find the angular velocity and acceleration of link 2. At the given instant OA= r, AB = s and $\theta = 45°$. (12)

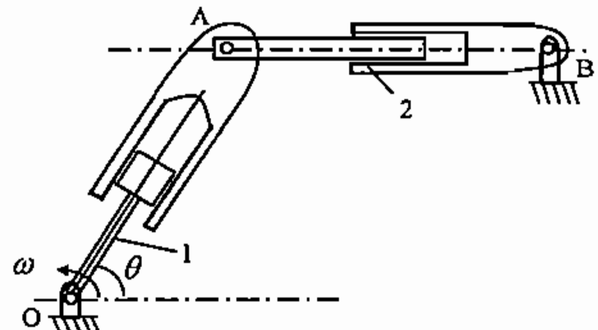

Fig. Q7

8. The force $\underline{F} = (x^2 + y\,t)\,\underline{i} + \cos t\,\underline{j} + t^2\,\underline{k}$ N acts on a material point which displaces from $\underline{r}_1 = 2\,\underline{j} + 3\,\underline{k}$ m at $t = 1$ s to $\underline{r}_2 = 4\underline{i} - 5\,\underline{k}$ m at t = 5 s with uniform velocity. Find: (i) position and velocity vector of the material point at time t, (ii) power of the force in terms of t. (6)

9. A thin rod of mass m is connected to O by a cable (Fig. Q9). It is released from initial position shown. Find $\ddot{\phi}$ and $\ddot{\theta}$ at the instant rod is released. (9)

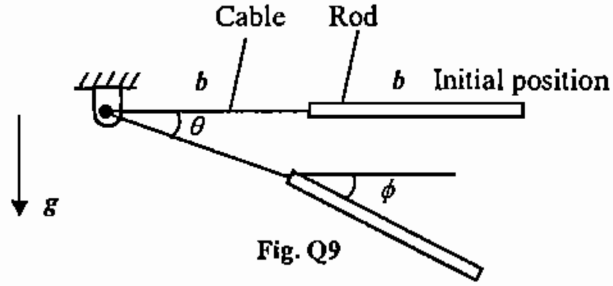

Fig. Q9

10. Show that for a conservative force field $\underline{F}$:

$$\underline{\nabla} \times \underline{F} = \underline{0}.$$ (4)

11. Derive the differential equations of equilibrium for a beam subjected to transversely distributed load q_y and distributed moment m. (5)

12. A thin rectangular plate (Fig. Q12) of mass m shown rotates at constant rate ω_2 with respect to arm AE, which itself rotates at constant rate ω_1 about Z axis. Neglecting gravity, find: (2, 2, 3, 7)
   (i) Mass moments of inertia at point A,
   (ii) Angular acceleration of the plate
   (iii) Acceleration of the mass centre of the plate
   (iv) Reaction forces and reaction moments at point A.

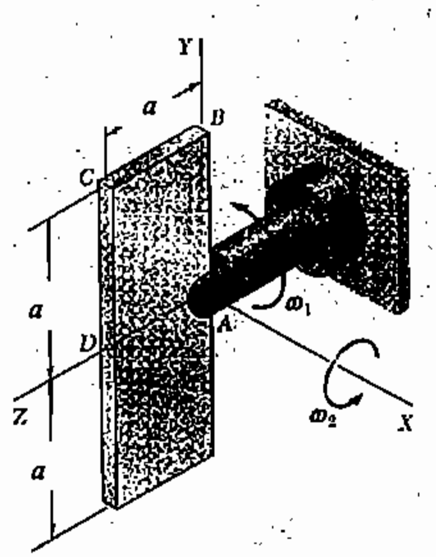

Fig. Q12