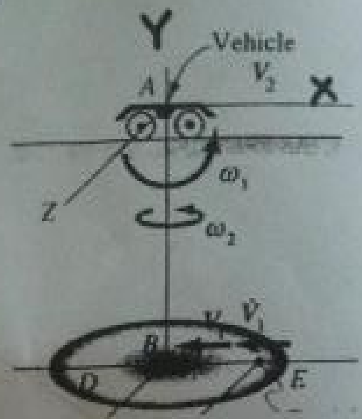
Q2 A conveyer system consists of a vehicle, which moves with a constant velocity V_2 to the right in the along a straight rail. A rod AB of Length L is mounted on the vehicle and swings with constant angular velocity ω_1 as shown. A circular disk D of radius R1 is mounted on the rod at its centre B and it spins at a constant angular velocity ω_2 with respect to the rod AB. On plate D, a particle has been given a motion V_1 and $\dot{V}_1$ (as shown) along a spoke BE which at this instant is parallel to the X axis. At the given instant the length BE = r.
Determine the velocity and acceleration of G with respect to a fixed set of axes in terms of the quantities given. Clearly indicate on which body the rotating axes have been mounted along with the location of their origin. (16)



FORMULA LIST

1) $\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z = \dot{s}\hat{e}_t$

2) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z = \ddot{s}\hat{e}_t + \dot{s}^2/\rho\,\hat{e}_n$

3) $E = mc^2$ 4) $I_z^A = I_z^C + m(x_c^2 + y_c^2)$

5) $I_{yz}^A = I_{yz}^C + m y_c z_c$ 6) $\vec{V}_P|_{XYZ} = \vec{V}_A + \vec{\omega} \times \vec{r}_{P/A} + \vec{V}_P|_{xyz}$

7) $\vec{a}_P|_{XYZ} = \vec{a}_A + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A}) + \dot{\vec{\omega}} \times \vec{r}_{P/A} + 2\vec{\omega} \times \vec{V}_P|_{xyz} + \vec{a}_P|_{xyz}$

8) $\rho = \dfrac{|\vec{V}|^3}{|\vec{V} \times \vec{a}|} = \dfrac{[1 + (\frac{dy}{dx})^2]^{3/2}}{|d^2y/dx^2|}$

9) $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + V(x)\psi = E(\psi)$

f) Cuboid: $\frac{m}{12}(a^2+b^2)$ etc...

10) $I_{AA} = \lambda_x^2 I_{xx} + \lambda_y^2 I_y + \lambda_z^2 I_z - 2\lambda_x\lambda_y I_{xy} - 2\lambda_y\lambda_z I_{yz} - 2\lambda_x\lambda_z I_{xz}$

Moments of inertia of some standard bodies
a) ring $(\frac{mR^2}{2}, \frac{mR^2}{2}, mR^2)$
b) disk $(\frac{mR^2}{4}, \frac{mR^2}{4}, \frac{mR^2}{2})$
c) rod $(\frac{mL^2}{12}, \frac{mL^2}{12}, 0)$
d) cylinder $(\frac{mR^2}{2}, m(\frac{R^2}{4}+\frac{L^2}{12}),)$
e) sphere $(\frac{2mR^2}{5}, \frac{2mR^2}{5},)$

11) $\vec{H}_A = \vec{H}_C + \vec{AC} \times m(\vec{V}_C - \vec{V}_A)$

12) $\{\vec{H}\} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$

13) $e = -\dfrac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$

14) $\vec{M}_A = (-I_{xz}^A\dot{\omega} + I_{yz}^A\omega^2)\hat{i} + (-I_{yz}^A\dot{\omega} - I_{xz}^A\omega^2)\hat{j} + I_{zz}^A\dot{\omega}\hat{k}$

15) $\vec{M}_A = \{I_{xx}^A\dot{\omega}_x - (I_{yy}^A - I_{zz}^A)\omega_y\omega_z\}\hat{i} + \{I_{yy}^A\dot{\omega}_y - (I_{zz}^A - I_{xx}^A)\omega_z\omega_x\}\hat{j} + \{I_{zz}^A\dot{\omega}_z - (I_{xx}^A - I_{yy}^A)\omega_x\omega_y\}\hat{k}$

16) $T = \frac{1}{2}mV_c^2 + \frac{1}{2}\{\bar{I}_{xx}\omega_x^2 + \bar{I}_{yy}\omega_y^2 + \bar{I}_{zz}\omega_z^2 - 2\bar{I}_{xy}\omega_x\omega_y - 2\bar{I}_{yz}\omega_y\omega_z - 2\bar{I}_{xz}\omega_x\omega_z\}$

AML 110
MECHANICS
Major Examination

24-11-2013

Max. Marks 120
Max. time: 120 + 5 minutes
Please answer all the questions in the answer book. Please read the full question paper before strating to answer. Numbers in parathesis indicate the maximum marks.

Q1 a) The position vector of a particle moving in the x-y plane in terms of polar cylindrical coordinates is ____ $\hat{i}_r$ + ____ $\hat{i}_\theta$. (2)

b) $\dfrac{d\hat{e}_r}{d\theta}$ = and $\dfrac{d\hat{e}_\theta}{d\theta}$ = (2)

c) A particle is moving in a planar path with its distance from the origin being given by $r = f(\theta)$. Express the velocity and acceleration of the particle in terms of f, $df/d\theta$, $d^2f/d\theta^2$, $\dot{\theta}, \ddot{\theta}$ and the unit vectors in polar cylindrical coordinates. (2+4)

d) Express the unit tangent vector $\hat{i}_t$ in terms of the quantities specified in c above. (2)

Q2) xyz axis are the coordinate axes and A is a point in the formulae no. 14 and 15 of the list

a) What are the conditions on the point A for both formulae to be valid?

b) What are the conditions on the xyz axes for the formula 1 to be valid?

c) What are the conditions on xyz axes for formula 2 to be valid? (2 x 3 = 6)

Q3 A toy is made by welding a thin rod of length R and mass M to a ring of radius R and mass M as shown.

a) Find the coordinates of centre of mass of the toy (1)

b) If the toy moves in plane motion on the ground with no slip with angular velocity ω and angular acceleration α (as shown in the figure), express the velocity and acceleration of A in terms of the given quantities. (2)

c) Consider the case when $\omega = 0$ but $\alpha \neq 0$. About which of the points A, B and D can we use the relation dH/dt = M. Give reasons for each of the answers. No marks will be awarded if reasons are not given. (3)

d) Consider the case when $\omega \neq 0$ and $\alpha \neq 0$. About which of the points of the body A, B and D can we use the relation dH/dt = M. Give reasons for each of the answers. No marks will be awarded if reasons are not given. (3)

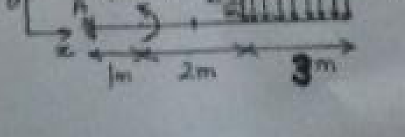
e) The system is released from a state of rest at t= 0 and at t=t_1 it reaches a configuration with point B at bottom and the rod BA vertically up with no slip and planar motion being maintained. Find the angular velocity at of the system at t= t_1. Gravity is in −y direction. (10)

A, B, D on body

Q4 A slender beam is fixed in the wall and loaded with a distributed force and a couple as shown.

a) Find the equivalent force along with the line of action for the distributed force.

b) Find the magnitude of shear force and bending moment at a section of the beam which is located at 2 m to the right of A. (10)



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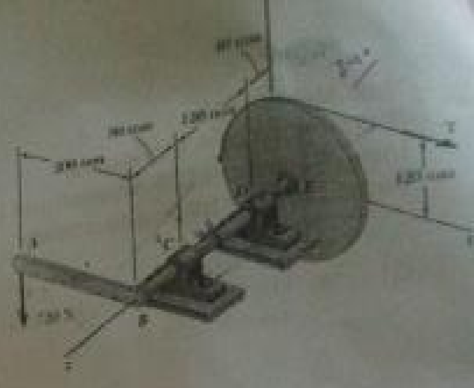
Q5. A 200-mm lever AB and a 240-mm diameter pulley are welded to the axle BE that is supported by bearings C and D. The axle BE is light and the weight of the disk and the lever is 1000 N each. If a 720 N vertical load is applied at A when the lever is horizontal determine

a) Tension T in the cord.

b) Reactions at C and D.

Assume that the bearing at D does not exert any axial thrust **and the system is in a static state** (12)



Q6. Consider the system of Q5. Assume $g = 10\ m/s^2$ so that the masses are simple numbers. At this instant when the system is at rest the string breaks. We wish to find the reactions and the angular acceleration (α) of the system at this instant. Carry out the following steps.

a) Draw a FBD of the entire rigid body (i.e. the disk, axle and the lever put together) showing the applied forces, weights and the reactions.

b) Find the location of centre of mass of the entire system.

c) Find I_z, I_{xz} and I_{yz} of the system about D (with axes being parallel to the given axes). Assume the lever to be a slender rod.

c) Find the acceleration of the centre of mass in terms of α.

d) Apply Newton's second law to the system to get 3 equations.

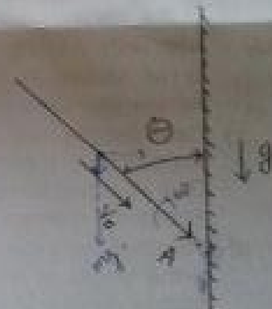
e) From the FBD find the Moments about point D of all the external forces.

f) Apply the proper version of Euler's equations to get the other 3 equations and thus solve for the unknown reactions and α. (15)

Q7. An arrow of mass m and length L is translating with a velocity V_0 as shown when it hits a smooth wall. After impact it moves such that the tip A has its velocity directed only along the wall (i.e. the arrow slides down the wall after impact).

a) Find the angular velocity of the arrow just after impact.

b) Find the impulsive reaction at the wall during the impact. (15)



Q8. A 5-kg slender rod AB is pin-connected to a 5-kg uniform disk as shown. Immediately after the system is released from rest determine the respective angular accelerations of the rod and the disc. (15)