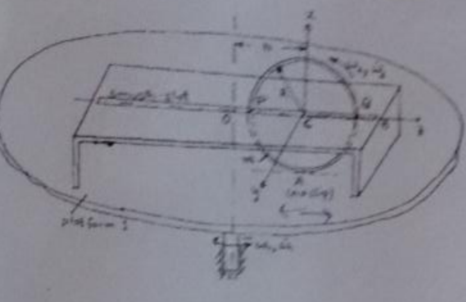
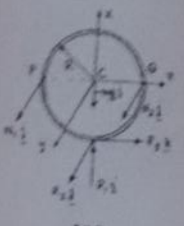




FBD

A thin ring of mass m and radius R rolls without slip on a point A on a rigid horizontal platform1 at angular velocity and angular acceleration $\omega_2\mathbf{j}$ and $\dot{\omega}_2\mathbf{j}$ respectively relative to platform 1. The platform rotates about the fixed vertical axis at $\omega_1\mathbf{i}$ and $\dot{\omega}_1\mathbf{i}$ relative to ground. The coefficient of friction at A is μ. The ring is constrained to move in a smooth radial slot OB on platform 1 with contacts at points P and Q. The FBD of the ring is shown with contact force (frictional and normal reactions) R_1, R_2 and R_3 at A and normal reactions from the slot N_1 and N_2 at P and Q respectively. Given m, R, b, ω_1, $\dot{\omega}_1$ and ω_2. The six unknowns are R_1, R_2, R_3, N_1, N_2 and $\dot{\omega}_2$.

a) Find $\bar{\omega}$, the angular velocity of the ring relative to the ground. [1]

b) Find $\dot{\bar{\omega}}$, the angular acceleration of the ring relative to the ground. [2]

c) Find the velocity of C, the centre of the ring, with respect to the platform. [1]

d) Find the acceleration of C with respect to the platform. [1]

e) Find the acceleration of C with respect to the ground. [4]

f) From the FBD drawn above compute, $\bar{M}_C$ the moment of all forces about C. [2]

g) Use Euler's equations for the ring to find $\bar{M}_C$ (the x, y and z components). [3]

h) Write explicitly 6 equations for R_1, R_2, R_3, N_1, N_2 and $\dot{\omega}_2$ in terms of the other given quantities. **Do not solve these equations.** [6]

i) Write an inequality containing R_1, R_2, R_3 and μ for the no slip condition to be valid. [2]

List of formulae

$$\bar{v}=\frac{d\bar{r}}{dt}=\dot{R}\bar{e}_R+R\dot{\theta}\bar{e}_\theta+\dot{z}\bar{k};\ \bar{a}=\frac{d\bar{v}}{dt}=\left(\ddot{R}-R\dot{\theta}^2\right)\bar{e}_R+\left(R\ddot{\theta}+2\dot{R}\dot{\theta}\right)\bar{e}_\theta+\ddot{z}\bar{k};\ \bar{v}=\dot{s}\bar{e}_t;\ \bar{a}=\frac{dv}{dt}\bar{e}_t+\frac{v^2}{\rho}\bar{e}_n$$

$$I^A_{zz}=I^C_{zz}+m\left(x_c^2+y_c^2\right);\ I^A_{yz}=I^C_{yz}+my_cz_c;$$

$$\text{Moments of inertia: Ring}\left(\frac{mr^2}{2},\frac{mr^2}{2},mr^2\right);\ \text{Rod}\left(\frac{ml^2}{12},\frac{ml^2}{12},0\right);\ \bar{v}_{P/XYZ}=\bar{v}_{A/XYZ}+\bar{\omega}\times\bar{r}_{P/A}+\bar{v}_{P/xyz}$$

$$\bar{a}_{P/XYZ}=\bar{a}_{A/XYZ}+\dot{\bar{\omega}}\times\bar{r}_{P/A}+\bar{\omega}\times\left(\bar{\omega}\times\bar{r}_{P/A}\right)+2\bar{\omega}\times\bar{v}_{P/xyz}+\bar{a}_{P/xyz};\ H_x=+\bar{I}_{xx}\omega_x-\bar{I}_{xy}\omega_y-\bar{I}_{xz}\omega_z$$

$$H_y=-\bar{I}_{yx}\omega_x+\bar{I}_y\omega_y-\bar{I}_{yz}\omega_z,\ H_z=-\bar{I}_{zx}\omega_x-\bar{I}_{zy}\omega_y+\bar{I}_z\omega_z$$

$$T=\tfrac{1}{2}m\bar{v}^2+\tfrac{1}{2}(\bar{I}_x\omega_x^2+\bar{I}_y\omega_y^2+\bar{I}_z\omega_z^2-2\bar{I}_{xy}\omega_x\omega_y-2\bar{I}_{yz}\omega_y\omega_z-2\bar{I}_{zx}\omega_z\omega_x);$$

$$I_{aa}=I_{xx}\lambda_x^2+I_{yy}\lambda_y^2+I_{zz}\lambda_z^2-2I_{xy}\lambda_x\lambda_y-2I_{yz}\lambda_y\lambda_z-2I_{zx}\lambda_z\lambda_x;\ \bar{H}_O=\bar{\bar{r}}\times m\bar{\bar{v}}+\bar{H}_C;$$

$$(M_A)_x=-I^A_{xz}\dot{\omega}+I^A_{yz}\omega^2;\ (M_A)_y=-I^A_{yz}\dot{\omega}-I^A_{xz}\omega^2;\ (M_A)_z=I^A_{zz}\dot{\omega};$$

$$M_x=\bar{I}_{xx}\dot{\omega}_x-\left(\bar{I}_{yy}-\bar{I}_{zz}\right)\omega_y\omega_z;\ M_y=\bar{I}_{yy}\dot{\omega}_y-\left(\bar{I}_{zz}-\bar{I}_{xx}\right)\omega_z\omega_x;\ M_z=\bar{I}_{zz}\dot{\omega}_z-\left(\bar{I}_{xx}-\bar{I}_{yy}\right)\omega_x\omega_y$$

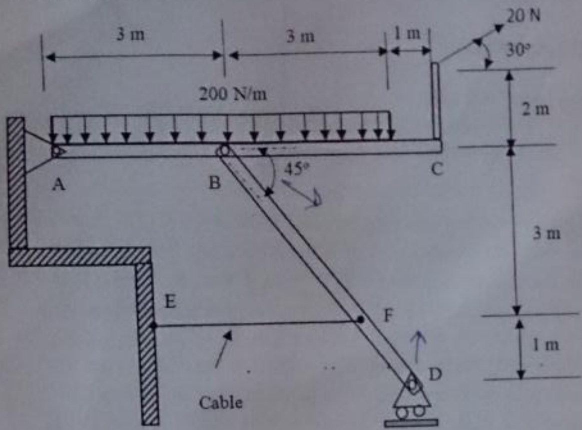
DEPARTMENT OF APPLIED MECHANICS, IIT DELHI

AML100 Mechanics (2013-2014- Second Semester)

Fri, 02 May 14, 13:00 - 15:00 (Time: 120+10 min)

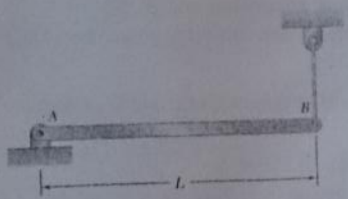
Major

1. For the Structure shown in Figure, determine the force in the cable EF. Neglect the self-weight of all the bars. [15]

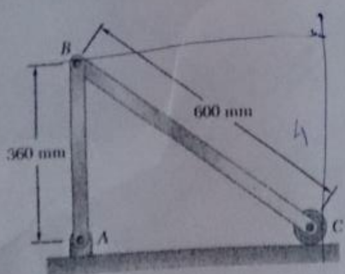


Q2. A uniform rod of length L and mass m is supported as shown in Figure below. If the cable attached at end B suddenly breaks, determine,

(i) the acceleration of end B [3]
(ii) the reaction at the pin support [3]
(iii) the shear force and bending moment at a section which is at a distance x from A immediately after the cable at B breaks [9]

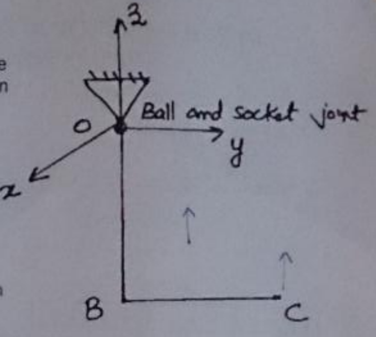


Q3. The uniform rods AB and BC of masses 2 kg and 4 kg, respectively, and the small wheel at C is of negligible mass (See Figure below). Knowing that in the position shown the velocity of wheel C is 2 m/s to the right, determine the velocity of pin B after rod AB has rotated through 90°. [15]



Q4. Consider an L shaped body OBC made by welding 2 slender rods OB and BC of mass m and length L each. Co-ordinate axis xyz are fixed to the body with origin at O as shown. To answer this question you can start from the expressions of moments and products of inertia of a slender rod.

a) Find the coordinates of centre of mass of OB, BC and hence the entire body OBC. [2]
b) Find I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} of the rod OB with respect to the given axes. [2]
c) Find I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} of the rod BC with respect to the given axes. [6]
d) Hence, find I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} of the entire body OBC with respect to the given axes. [2]
e) Find I_{kk} for the entire body OBC about an axis k-k passing through OC. [6]



Q5. The L shaped body of the previous question now hangs from the wall by a ball and socket joint at O as shown. Initially the body is at rest. A bullet of mass m_0 travelling with a velocity $(-V_0 \boldsymbol{i})$ hits the body and gets embedded in it at C. If the post impact angular velocity of the body and the bullet is $\omega_x \boldsymbol{i} + \omega_y \boldsymbol{j} + \omega_z \boldsymbol{k}$, set up three scalar equations to find ω_x, ω_y and ω_z in terms of the given parameters. **Do not solve these equations.** [15]

Q6. Shaft BC is pinned to a T-bar, which rotes at a constant angular speed ω_1 with respect to the vertical axis fixed to the ground as shown. Wheel C rotates freely relative to shaft BC. The platform, over which wheel C rolls, is raised at a constant speed u, causing the angle β to decrease. The wheel does not slip relative to the platform in the direction transverse to the diagram, but slipping in radial direction is observed to occur. The angular velocity of the wheel with respect to ground can be expressed as $\vec{\omega}_1 + \dot{\beta}\mathbf{k} + \omega_3\mathbf{i}$, where ω_3 is the angular velocity of the wheel with respect to the shaft BC. We wish to find $\dot{\beta}$ and ω_3 in terms of the other given parameters.

a) Express $\vec{\omega}_1$ in terms of the given unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. [1]
b) Find the velocity of B with respect to ground frame in terms of the given parameters. [1]
c) Find the velocity of C with respect to ground frame in terms of the given parameters and $\dot{\beta}$. [2]
d) Find the velocity of D (the point on the wheel in contact with the platform) with respect to ground frame in terms of the given parameters, $\dot{\beta}$ and ω_3. [3]
e) Find $\dot{\beta}$ and ω_3 by comparing the transverse and vertical components of velocities of D and D' (a point on the platform coincident with D). [4]
f) Find $\ddot{\beta}$ and $\dot{\omega}_3$ by taking a time derivative of expressions obtained in e) above. [4]
g) Let the angular acceleration of the wheel with respect to ground be $A\boldsymbol{i} + B\boldsymbol{j} + C\boldsymbol{k}$. Write the expressions for A, B and C in terms of the given quantities, $\dot{\beta}$, ω_3, $\ddot{\beta}$ and $\dot{\omega}_3$. [5]

We leave this problem here. Next year the students will be asked to apply Euler's equations to the wheel to find the contact forces at D.