APL 100

Major Exam, Sep 5, 2020, Full Marks: 40

Guidelines: Open-book. Give all formulae used and **explain your steps** in each problem.

Instructions:

- The question paper will be uploaded on course website (and also emailed to your IITD email) at 11 AM on 5th September. The answer script MUST be uploaded on GRADESCOPE latest by 1:30 PM the same day. WHATSAPP submission will NOT be accepted.
- Each question should start on a fresh page and must be labelled on GRADESCOPE to their corresponding pages. We will NOT consider your submission if you do not label your PDF pages on GRADESCOPE.
- Only PDF files must be submitted on GRADESCOPE. If you submit image files, they will NOT be graded.
- Late submissions will not be accepted. It is also important that you mention/label pages corresponding to different questions in GRADESCOPE before 1:30 PM. You won't be able to label them after 1:30 PM.
- The answer script must be in your own handwriting else it will be rejected. Also, if two persons are found to have same solution method, their answer script will also be rejected.
- You will have to write on paper (please use A4 size papers) in your own handwriting. So, keep 10-15 pages of A4 size papers ready with you.
- You must write your name, entry number and page number on each sheet.
- You must make sure that the handwriting is neat and scan is clear.

Problem 1: Look at the system shown in Figure 1 where a straight bar CD is connected to a curved beam ADB through a collar joint at D. An external moment of magnitude M and perpendicular to the plane of paper (outward) acts at the end A. Assume gravity to be absent. (**5 points**)

(a) Assuming friction acts between the collar and the curved beam, draw free body diagram of both the curved beam and the bar CD separately. Is it a statically determinate system?
(b) Suppose the collar is frictionless. Solve for all the reaction forces and moments at B, C and D.

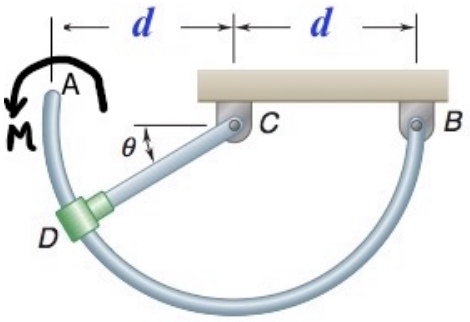

Figure 1

Problem 2: A ball of mass m is moving in a circular motion in a horizontal plane as shown in Figure 2. The ball is supported through an inextensible massless string which makes an angle θ from the vertical line. Suppose the string is then pulled from the support end so that the length of the string between support and ball starts to decrease at a constant rate of η (for negative η, the length increases). Assume gravity acts vertically downward. (**10 points**)

(a) Obtain the acceleration of the ball the moment the string starts to be pulled. Use cylindrical coordinate system whose origin is at the support and its z axis is vertically upward.
(b) What would be the tension in the cable just before and after the string is started to be pulled.
(c) In which case, tension in the string will be higher: when η is positive or negative?

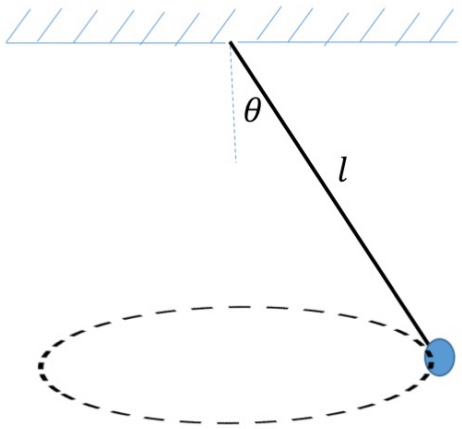

Figure 2

Problem 3: Consider a rigid body of arbitrary shape which is attached at its mass center O and subjected to no force other than its weight and reaction force from the support at O.(**10 points**)

(a) Prove that the angular momentum $\vec{\mathrm{H}}_O$ of the body about its mass center is constant in magnitude and direction, that the kinetic energy T of the body is constant, and that the projection along angular momentum vector of the angular velocity $\vec{\omega}$ of the body is constant.
(b) Show that the tip of vector $\vec{\omega}$ (when its other end is placed at the origin) describes a curve on a fixed plane whose normal lies along $\vec{\mathrm{H}}_O$ and at a distance of $2T/H_O$ from O. Here H_O is the magnitude of angular momentum.
(c) Show that with respect to a frame of reference attached to the body and its axes coinciding with the principal axis of inertia, the tip of the vector $\vec{\omega}$ ((when its other end is placed at the origin)) appears to describe a curve on an ellipsoid. Obtain the equation of this ellipsoid.

Problem 4: A uniform solid spherical ball of mass m, radius r is rolling without slipping with its center of mass velocity $v\ \hat{i}$. It then collides inelastically with the corner of a step of height h ($< r$) as shown in Figure 3. The points C, O are in the x-y plane as shown in the figure. Assume that no slipping occurs at the impact point and that the rotation of the sphere remains about negative z axis (even after collision). Assume gravity acts along negative Y axis. (**10 points**)
(a) Draw the free body diagram of the sphere during collision.
(b) If you consider the same coordinate system but with its origin at O, find out by visual inspection which among the three axes (x, y, z) are principal axis for the sphere.
(c) Write the angular momentum equation (Euler's second axiom) about point 'O' w.r.t the inertial frame.
(d) Find in terms of h and r, the minimum speed v for which the ball will trip over the step.

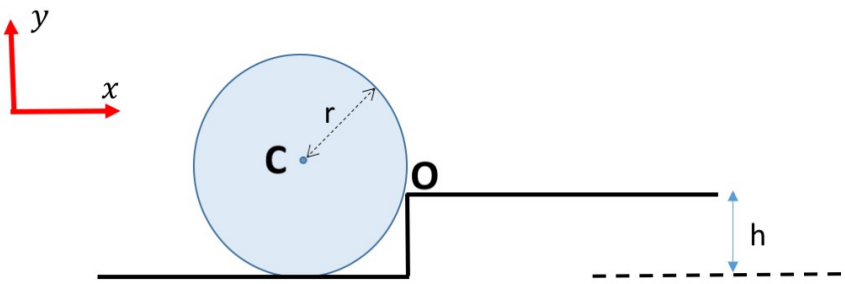

Figure 3

Problem 5: Determine the external torque M which must be applied at O in order to support the mechanism in the position shown. The masses of the disk at C, bar OA, and bar BC are m_o, m, and $2m$ respectively. Gravity is assumed to act downward. (**5 points**)

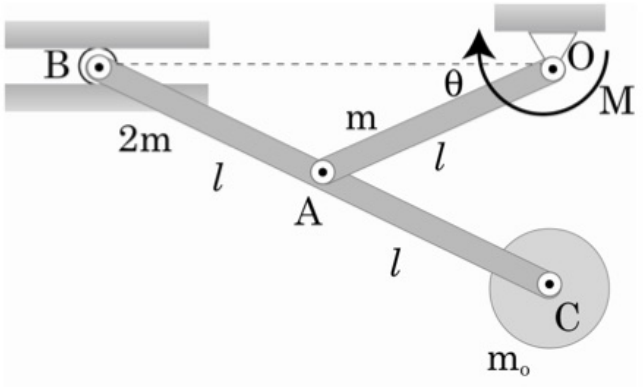

Figure 4