APL100 MECHANICS MINOR I **PART B (30 marks)**

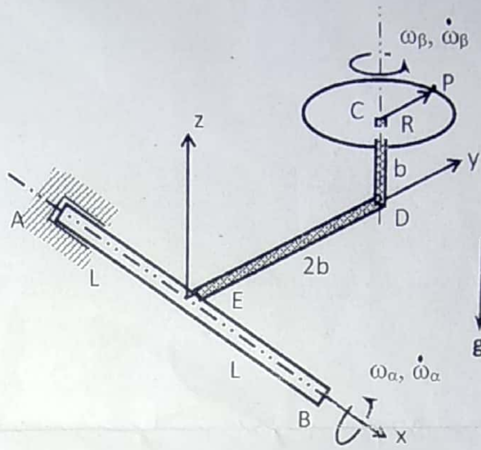
6th February 2018 11.30 am to 12.00 pm

Q1: Derive the following relationship between the angular momentum of a rigid body of mass m about any point A in space and the angular momentum of that rigid body about its centre of mass C with respect to the fixed frame F:

$\vec{H}_{A|F} = \vec{H}_{C|F} + \vec{r}_{CA} \times m\,\vec{V}_{CA|F}$, starting from $\vec{H}_{A|F} = \int \vec{r}_{PA} \times \vec{V}_{PA|F}\,dm$

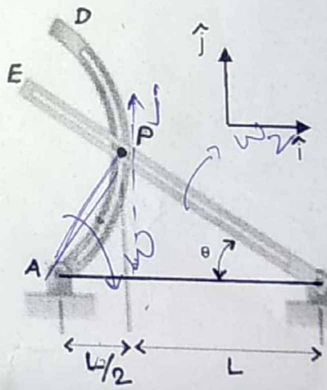
How does this relation change if F is inertial frame ? (4+1)

Q2: A circular disc of radius R and mass m is rotating at an angular velocity ω_β and angular acceleration $\dot{\omega}_\beta$ relative to a light bent rod CDE. Rod CDE is **rigidly attached** to another light rod AEB at E as shown. Rod AEB rotates at an angular velocity ω_α and angular acceleration $\dot{\omega}_\alpha$ relative to the ground about axis AB. Find the velocity and acceleration at point P with respect to the ground frame, where CP is parallel to y-axis.



Length AE = Length EB = L; Length ED = $2b$, length CD = b and length CP = R. (15)

Q3: The motion of pin P is guided by slots cut in rods AD and BE. Bar AD has a constant angular velocity ω_1 rad/s **clockwise**. Bar BE has an angular velocity ω_2 rad/s **counterclockwise** and is slowing down at a rate α rad/s^2. Determine the velocity of pin P for the position shown with respect to the ground frame.



tangent at P is along $\hat{j}$

$\sqrt{\frac{L^2}{4}+L^2}$

$5L^2$

$\frac{\sqrt{5}L}{2}$

Length AF = $L/2$; Length FB = L; $< PBF = \theta = 45^o$. (10)

Formula Sheet for Minor I

Velocity and Acceleration in Cartesian coordinates

$$\vec{v}(t) = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}, \quad \vec{a}(t) = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

Velocity and Acceleration in Cylindrical polar coordinates

$$\vec{v}(t) = \dot{r}\hat{e}_r + r\dot{\phi}\hat{e}_\phi + \dot{z}\hat{e}_z, \quad \vec{a}(t) = (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{e}_\phi + \ddot{z}\hat{e}_z$$

Velocity and Acceleration in path coordinates

$$\vec{v}(t) = \dot{s}\hat{e}_t, \quad \vec{a}(t) = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

Angular velocity of a rotating frame and derivative of an aritrary vector

$$\vec{\omega} = \frac{1}{2}(\hat{e}_i \times \dot{\hat{e}}_i), \quad \dot{\vec{A}}_{|F} = \dot{\vec{A}}_{|m} + \vec{\omega} \times \vec{A}$$

Composition of angular velocity and angular acceleration

$$\vec{\omega}_{3|1} = \vec{\omega}_{3|2} + \vec{\omega}_{2|1}, \quad \dot{\vec{\omega}}_{3|1} = \dot{\vec{\omega}}_{3|2} + \dot{\vec{\omega}}_{2|1} + \vec{\omega}_{2|1} \times \vec{\omega}_{3|2}$$

Expressions of Velocity and Acceleration

$$\vec{v}_{P|F} = \vec{v}_{A|F} + \vec{v}_{P|m} + \vec{\omega} \times \vec{r}_{PA}$$

$$\vec{a}_{P|F} = \vec{a}_{A|F} + \vec{a}_{P|m} + \dot{\vec{\omega}} \times \vec{r}_{PA} + 2\vec{\omega} \times \vec{v}_{P|m} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{PA})$$

Euler Axioms

$$\dot{\vec{p}}_{|I} = \vec{F}_R, \quad \dot{\vec{H}}_{O|I} = \vec{M}_O$$

Relation of moments at different points:

$$\vec{M}_B = \vec{M}_A + \vec{r}_{AB} \times \vec{F}$$

Euler second axiom about an arbitrary point:

$$\dot{\vec{H}}_{A|I} = \vec{M}_A - \vec{r}_{CA} \times m\vec{a}_{A|I}$$