APL100-Minor Exam: 27th Dec, 2020, 11:30 am – 1 pm

Instructors: Sitikantha Roy, Amitabh Bhattacharya, Department of Applied Mechanics

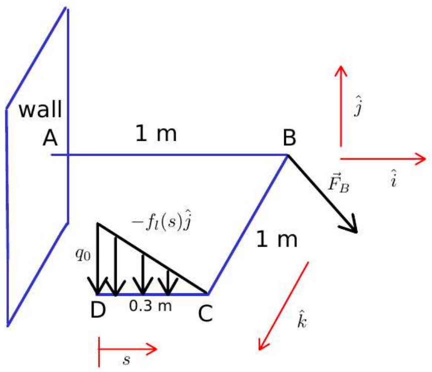
Total points: 30

Instructions

1. This minor will be released at 11:30 am on 27th Dec and will be closed at 1 pm on the same day.
2. It should take around 1 hour to attempt the exam. An additional 30 mins is being provided to account for network issues, as well as scanning the exam.
3. You have to scan and upload your answersheets on gradescope latest by 1 pm – there is no grace period for uploading the answer sheets. Your answersheet will not be graded if we recieve it after the closing time.
4. During the exam, please point your laptop/mobile camera towards yourself (showing sideview), and stay connected to the google meets link for your section.
5. You are NOT supposed to access the internet during minor, except to download the QP and upload the scanned answer sheet
6. Any copying/collaboration between students (e.g. in person or via whatsapp or other social media/VC platforms/email etc) is strictly prohibited. Any student(s) caught using such unfair means will be a awarded a minimum of letter grade penalty, in addition to receiving zero marks for the exam
7. Make sure to mention your name and entry number in the first page
8. Please carefully indicate the page numbers for each question on gradescope after uploading the pdf
9. This is a subjective type exam. Please state clear justification and steps for your answers. Simply stating the final answer for any question or its sub-part will NOT fetch you any points.

Best of luck for the exam !

Q1 (6=3+3): Consider the rigid frame ABCD anchored to the wall at point A. Cross sectional area/thickness of the beams in the frame can be neglected for this problem. Note that AB and DC are oriented along $\hat{i}$, while BC is oriented along $\hat{k}$. Lengths *l(AB)*=*l(BC)*=1 m, while length *l(DC)*=0.3 m. A force $\vec{F}_B = 2\hat{j}$ acts at point B, and a line force distribution $-f_l(s)\hat{j}$ acts along DC, where $f_l(s) = (0.3 - s)q_0$, and $q_0 = 1$ N/m. Here s is the distance from D.
(a) Find net force $\vec{F}_R$ and moment $\vec{M}_A$ at point A *only* due to point force $\vec{F}_B$ and line force distribution $-f_l(s)\hat{j}$.
(b) Assuming A is the origin of the coordinate system, find the (*x*,*z*) location of the simplest force system on the $y = 0$ (i.e. $x - z$) plane. (Hint: Simplest force system should have zero net moment at this location for the given configuration).

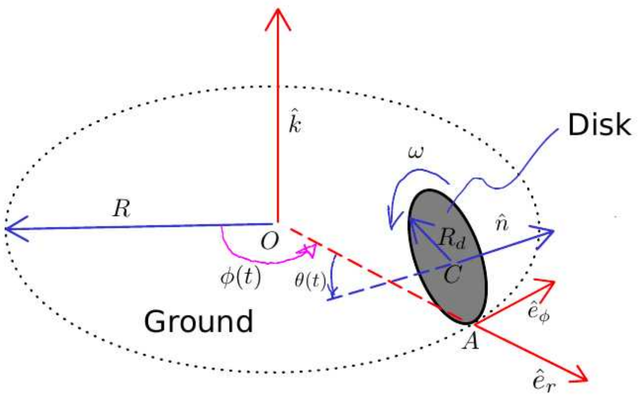


Q2. (3+5+4): A thin circular disk (center *C*) rolls without slip on flat ground, so that the point of contact *A* always lies on a circular trajectory with radius *R=1* m and center *O*, where point *O* lies on the flat ground. Radius of disk is $R_d = 0.2$ m. The instantaneous radial unit vector $\hat{e}_r$ (oriented along $\vec{OA}$) and instantaneous azimuthal unit vector $\hat{e}_\phi$ (tangent to circle) have been shown, along with the $\hat{k}$ axis. The point *C* lies in the *r-z* plane at all time, and the instantaneous unit normal to the disk $\hat{n}$ subtends an angle $\theta(t)$ with respect to $\hat{e}_r$ as shown in the figure. Here $\hat{n}$ is always coplanar with $\hat{e}_r$ and $\hat{k}$. At the given instant in time, t, $\theta = \pi/4$ rad, and $\dot{\phi} = 1$ rad/s.

We can define the following reference frames here. Ground frame G; Frame 1 (dummy): Given by $\hat{e}_r$, $\hat{e}_\phi$; Frame 2 (dummy): Given by $\hat{n}$, $\hat{e}_\phi$; Frame 3: Fixed to the disk

Assuming $\ddot{\theta} = \ddot{\phi} = 0$, and neglecting thickness of the disk:
(a) Use no-slip condition to find the value of magnitude of angular velocity, ω, of the disk (Frame 3) with respect to Frame 2.
(b) Find angular velocity vector and angular acceleration vector of disk <u>with respect to ground frame</u> if $\dot{\theta} = 0$
(c) Find velocity vector of *C*, $\vec{v}_C$, and acceleration vector of *C*, $\vec{a}_C$, if $\dot{\theta} = 1$ rad/s

<u>All the vectors should be expressed in terms of $\hat{e}_r$, $\hat{e}_\phi$ and $\hat{k}$</u>



Q3. (2+4+4+2): A slotted arm OA rotates about the horizontal axis at O in the Figure below, with $\dot{\theta} = -2$ rad/s and $\ddot{\theta} = 1$ rad/s^2 at $\theta = 30°$. A pin (denoted by P, with circular cross section) of mass *m*=0.2 Kg can slide in the linear slot and is kept in contact with the surface of fixed disc/circular plate due to the spring force. The pin is also in contact with only one of the slot walls in the given position. Assume the pin slips with respect to the disc surface and the slot wall. Length of OA=0.8 m and gravity *g=9.8* m/s^2 acts along $-\hat{j}$. No friction force is present between the pin and the surface of the fixed disc/circular plate. A friction force is however present between the slot walls and the pin.

3.1. What is the angular velocity vector and angular acceleration vector of arm OA ? Also express $r,\ \dot{r}, \ddot{r}$ in terms of $\theta,\ \dot{\theta},\ \ddot{\theta}$ (here $\hat{e}_r$ points along OA as shown in Figure, and $\hat{n} = \hat{e}_r \times \hat{e}_\theta$).

3.2. Draw the **free body diagram** of the pin P**.** (Showing all the forces, including reaction and friction forces, acting on the pin)

3.3. Find $\vec{v}_P$ and $\vec{a}_P$ (Velocity and Acceleration of pin P) in terms of $\hat{e}_r$ and $\hat{e}_\theta$

3.4. Find the normal components of the reaction forces from the disc surface and the slot wall **on the pin**. The free length of the spring is $l_0 = 1$ m and spring constant is *k*=100N/m. The coefficient of dynamic friction for the pin sliding on slot wall is μ_k=0.25

Additional information: a. The magnitude of spring force on pin along $\vec{AP}$ is given by $F_s = -k.(l - l_0)$, where $(l - l_0)$ is the extension of the length of the spring (l) over its free length (l_0) b. Recall that dynamic friction force $\mu_k.N$ **on pin** acts opposite to the direction of the relative velocity of the pin with respect to slot wall, where *N* is the magnitude of the net normal reaction force from the slot wall in this case. c. Angle subtended by the diameter at any point on the circumference, i.e. OPB =90 degree