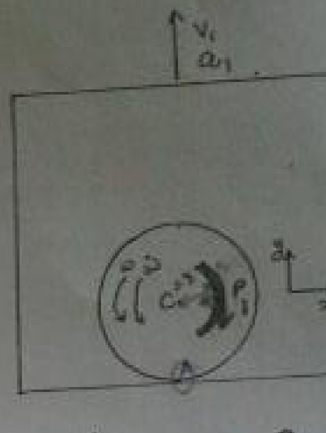
Q4. An elevator is moving up with velocity $v_1 = 10$ m/s and acceleration $a_1 = 2$ m/s^2 as shown. A cylinder of radius $r = 1$ m rolls without slip on the floor of the elevator with angular velocity $\omega = 2$ rad/s and angular acceleration $\dot{\omega} = 1$ rad/s^2 as shown. The cylinder has a circular slot of radius 0.8 m which is also centred at C, the centre of the cylinder. A pointer P moves with a constant downward speed 1m/s and is at the location as shown (CP is along the x axis).

a) Find the velocity and acceleration of C with respect to a ground frame of reference.

b) Find the velocity and acceleration of P with respect to a frame mounted on the cylinder.

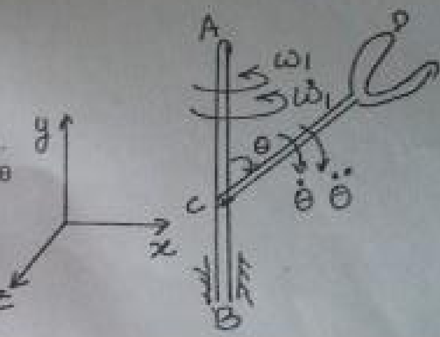
c) Find the velocity and acceleration of P with respect to a ground frame of reference. (2+3+10 marks)



Q5. A straight bar AB is rotating with angular velocity ω_1 and angular acceleration $\dot{\omega}_1$ with respect to a ground frame as shown. On the rod AB an antenna CDE is mounted which makes an angle θ with the AB axis and changes at the rates $\dot{\theta}$ and $\ddot{\theta}$ with respect to the rod AB. Determine:

a) The angular velocity of the antenna with respect to a ground frame of reference.

b) The angular acceleration of the antenna with respect to a ground frame of reference. (2+4 marks)



Some formulae: (may or may not be needed)

1) $\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z = \dot{s}\hat{e}_t$

2) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z = \ddot{s}\hat{e}_t + \dot{s}^2/\rho\,\hat{e}_n$

3) $E = mc^2$ 4) $I_z^A = I_z^C + m(x_c^2 + y_c^2)$

5) $I_{yz}^A = I_{yz}^C + m y_c z_c$ 6) $\vec{V}_P|_{XYZ} = \vec{V}_A + \vec{\omega} \times \vec{r}_{P/A} + \vec{V}_P|_{xyz}$

7) $\vec{a}_P|_{XYZ} = \vec{a}_A + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A}) + \dot{\vec{\omega}} \times \vec{r}_{P/A} + 2\vec{\omega} \times \vec{V}_P|_{xyz} + \vec{a}_P|_{xyz}$

8) $\rho = \frac{|\vec{V}|^3}{|\vec{V} \times \vec{a}|} = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

9) $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E(\psi)$

f) Cuboid: $\frac{m(a^2+b^2)}{12}$ etc..

10) $I_{AA} = \lambda_x^2 I_{xx} + \lambda_y^2 I_{yy} + \lambda_z^2 I_{zz} - 2\lambda_x\lambda_y I_{xy} - 2\lambda_y\lambda_z I_{yz} - 2\lambda_x\lambda_z I_{xz}$

Moments of inertia of some standard bodies

a) ring $(\frac{mR^2}{2}, \frac{mR^2}{2}, mR^2)$

b) disk $(\frac{mR^2}{4}, \frac{mR^2}{4}, \frac{mR^2}{2})$

c) rod $(\frac{mL^2}{12}, \frac{mL^2}{12}, 0)$

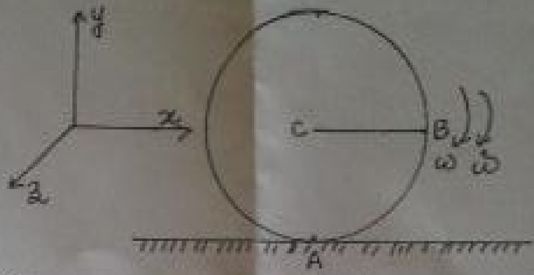
d) cylinder $(\frac{mR^2}{2}, m(\frac{R^2}{4}+\frac{L^2}{12}),)$

e) sphere $(\frac{2mR^2}{5}, \frac{2mR^2}{5},)$

Maximum Marks : 60 7-10-2013

Please answer all the questions. Please read the full question paper before you start answering. You can answer the questions in any order but all the working for the same question should be done together. **The answers for all vector quantities must be expressed either in terms of appropriate unit vectors or with the proper direction. Do not forget units in case of numerical answers.**

Q1. Consider a homogeneous ring of radius R and mass m with centre at C. The ring is welded to a homogeneous, slender rod BC of length R and mass m.



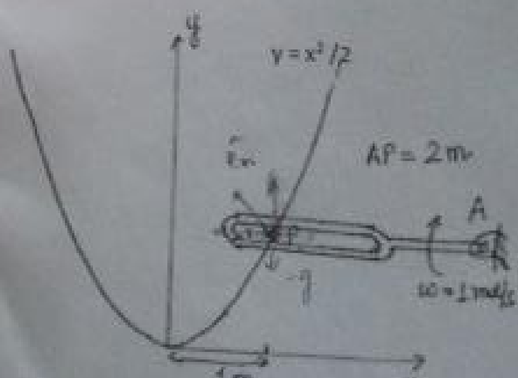
The angular velocity and angular acceleration of the body is as shown in the figure. The composite ring rolls without slip on the ground.

a) Find the velocity of points A, C and B.
b) Find the acceleration of points C, A and B.
c) Find the radius of curvature of the path of the points C and B. (2+3+5 marks)

Q2. Consider the composite body of Q1. Determine the following moments and products of inertia for this body. You can start from the expressions for the inertia terms for a ring and slender rod.

a) I_x and I_{xy} with the origin of the axes at C.
b) I_x, I_y, I_z and I_{xy} with the origin of the axes at A.
c) I_k where k is an axis passing through B and A. (2+8+4 marks)

Q3. A particle P of mass 1 kg moves along a parabolic path $y = 0.5\,x^2$ and is at a location x = 1m. It is also constrained to move in the slotted bar which at the instant shown is rotating with a constant angular velocity ω = 1 rad/s (clockwise). The gravity vector is in the $-\hat{j}$ direction. At the given instant, when AP = 2m and is parallel to the x axis as shown in the figure.



Determine the following:

a) the unit tangent vector $\hat{e}_t$ and the unit normal vector $\hat{e}_n$ of the path of the particle,
b) the radius of curvature, ρ, of the path of P,
c) the velocity of the particle P,
d) the acceleration of the particle P, and
e) the normal reactions N1 and N2 exerted by the parabolic path and the slotted bar respectively, on the particle P. Assume that friction at all contacts is negligible.

(2+2+3+5+3 marks)