APL 100: Minor 2

Full Marks: 40   Duration: 1 hr   Start time: 10:00 AM   Date: 16^{th} Aug 2020

Instructions:

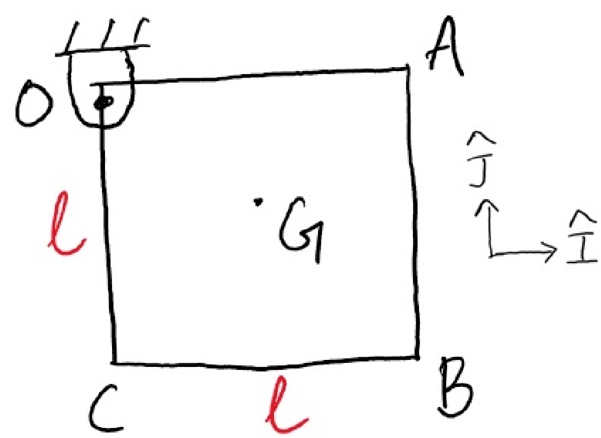
- It is an open book/open notes exam. You can use whatever resources you want to prepare your solution but you are not supposed to discuss with each other. The answer-script must be in your own handwriting else it will be rejected. Also, if two persons are found to have same solution method, their answerscripts will also be rejected.
- Please use A4 size paper to write your solution and please write clearly so that it is readable to us.
- Write the solution of a new question in a fresh page and make sure to write your name and entry number on top of each page.
- When you upload your scanned document to GRADESCOPE, do not forget to label your pages, i.e, the corresponding pages of each question.
- Make sure the scan is of good quality, i.e, look at your own scanned document on a computer to make sure it is readable.
- The scanned answerscripts must be uploaded on GRADESCOPE by 11:20 AM, i.e., within 20 minutes of exam getting over.
- In case of difficulty in uploading on GRADESCOPE due to poor connectivity, please whatsapp your scanned pages latest by 11:20 AM. Eventually, the scanned pages must be uploaded on GRADESCOPE latest by 11:30 AM but it must be the same copy which you whatsapped by 11:20 AM.
- Use the following Whatsapp number for sending your answerscript: 08586972599

Problem 1: We have learnt in the class the angular momentum balance equation in an inertial frame for a rigid body about its mass center (c), i.e,

$$\frac{d}{dt}(\underset{\sim}{\mathrm{I}}^c\ \vec{\omega}_{\mathcal{R}/\mathcal{I}})_{/\mathcal{I}} = \vec{\mathcal{M}}^{ext}_{/c}. \tag{1}$$

(a) Derive that $\vec{\alpha}_{\mathcal{R}/\mathcal{I}} = \frac{d}{dt}(\vec{\omega}_{\mathcal{R}/\mathcal{I}})_{/\mathcal{R}}$ where $\vec{\alpha}_{\mathcal{R}/\mathcal{I}}$ is the angular acceleration of the rigid body relative to inertial frame. (**3 points**)
(b) Derive the final vector form of angular momentum balance equation in terms of angular velocity and angular acceleration. (**3 points**)
(c) Show that in a coordinate system fixed to the rigid body, the components of angular acceleration and angular velocity are related in a simple way, i.e., $\alpha_i = \dot{\omega}_i$. Here the subscript i is used to denote the component of the corresponding vector along the ith axis of the coordinate system. (**3 points**)
(d) Finally obtain the three components of the vector equation in (b) above in a coordinate system attached to the rigid body and with its three axes lying along the principal axis of inertia of the rigid body. (**6 points**)

Problem 2: A square plate OABC of mass m is hinged (2D loading) at one of its corners O as shown in the figure. The plate is released from rest in the position shown. G is the center of mass of the plate. Assume planar motion and that gravity acts along $-\hat{J}$.
(a) Draw the free body diagram of the plate and write the equations of motion (both linear momentum and angular momentum balance equations) of the plate. (**3 points**)
(b) Find the reaction force at O immediately after release. (**3 points**)
(c) Find the angular velocity of the plate when OA makes an angle θ with $\hat{I}$ axis, using work energy theorem. (**4 points**)



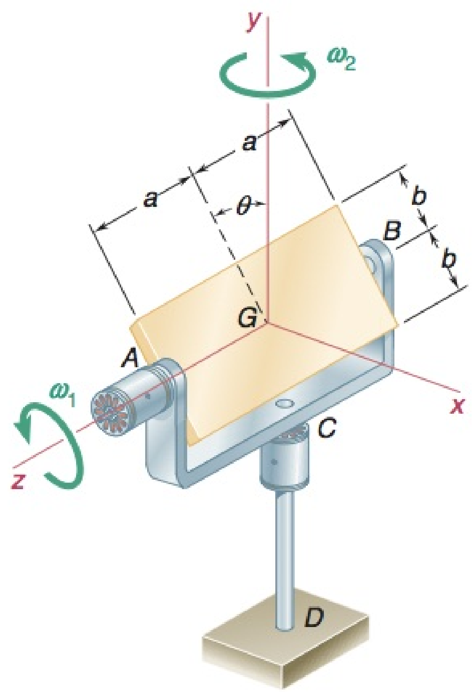
: Figure of Problem 2

Problem 3: An advertising panel of mass M (shown in figure below), length $2a$ and width $2b$ (having negligible thickness) is kept rotating at a constant rate ω_1 about its horizontal axis by a small electric motor attached at A to frame ACB. The frame itself is kept rotating at a constant rate ω_2 about a vertical axis by a second motor attached at C to the column CD. Assume the support at D to be part of an inertial frame, e.g., ground. Further assume that the frame ACB, the motors and column CD are all massless. Also neglect gravity. Express all the quantities (asked below) in the xyz coordinate system which is fixed to frame ACB as shown in figure.

(a) Write down the expression of angular momentum of the advertising panel about point 'G' w.r.t to the inertial frame. (**4 points**)

(b) Write down the expression of rate of change of angular momentum (obtained in (a)) relative to both the frame ACB as well as the inertial frame. (**6 points**)

(c) Express in terms of angle θ the dynamic reaction force and reaction moment (if any) exerted on column CD by the support at D. (**5 points**)



: Figure of Problem 3