Marks 120 MAJOR TEST Sr. No Entry No

MECHANICS OF SOLIDS AM140 Time 2h 10 min

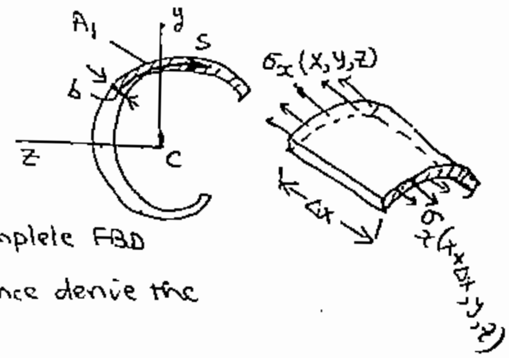
(2) 1. For alternating stress σ_a with zero mean, failure occurs after N cycles: $N = \left(\frac{\sigma_u - \sigma_e}{\sigma_a - \sigma_e}\right)^q$. A bar is subjected to alternating stress σ for n cycles. Set up an equation to find the subsequently applied alternating stress σ_1 so that it fails after n more cycles.

(6) 2. For axisymmetric problem of thick cylinder under internal and external pressure: $u_r = u_r(r)$, $u_\phi = 0$, $\sigma_r = \frac{\Psi}{r}$, $\sigma_\phi = \Psi'$, $\sigma_z = \sigma_{r\phi} = \sigma_{\phi z} = \sigma_{zr} = 0$ where $\Psi' = \frac{d\Psi}{dr}$ and $\Psi(r)$ is stress function. Young's modulus is E and Poisson's ratio is ν. Derive the compatibility equation in terms of Ψ.

(10) 3. For beam bending with arbitrary axes, under bending

$$\sigma_x = \frac{-y\,(I_{yy}M_z - I_{yz}M_y) + z\,(I_{zz}M_y - I_{yz}M_z)}{I_{yy}I_{zz} - I_{yz}^2}$$

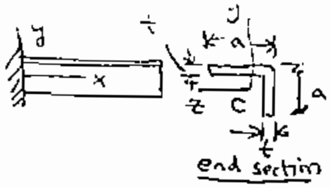
For the hatched area A_1: $Q_z = \int_{A_1} y\,dA$, $Q_y = \int_{A_1} z\,dA$. Complete FBD of element shown. Set up its equilibrium equation and hence derive the expression for σ_{sx}^{av}.



(5) 4. A circular bar is subjected to bending moment M and twisting moment T. Prove that the equivalent bending moment M_e which acting alone by itself produces the same maximum tensile stress is given by $M_e = \frac{1}{2}M + \frac{1}{2}\sqrt{M^2+T^2}$.

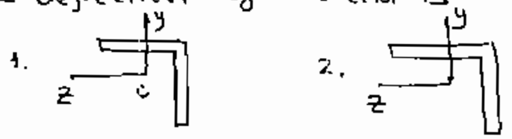
4) 5. A beam of length L and circular cross-section is subjected to bending moment $M = M_1 \frac{x^2}{L^2}$. Express its strain energy in the form $U = \alpha \frac{\sigma_{max}^2}{2E}$ (volume) to find α.

6. A cantilever of thin angle section is subjected to tip load P.



(1) (a) Mark the position of the tip load in the end section shown for which the cantilever bends without twisting.

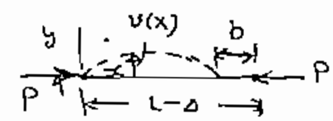
(1+2) (b) Mark the two possible lines of action of end load P on the end section so that the deflection of the end is in the direction of the load.



(8) 7. For thermal stresses in beams, using $u(x,y) = u_0(x) - y v'$, $v = v(x)$, prove that

$$\delta U = \int_0^L (-N'\delta u_0 + M''\delta v)\,dx + (N\delta u_0 + M\delta v' - M'\delta v)\Big|_0^L .$$

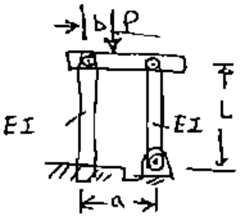
(4) 8. Prove that the axial displacement Δ due to bending of axially loaded bar by deflection $v(x)$ is $\simeq \frac{1}{2}\int v'^2 dx$.



(12) 9. For simply-supported bar subjected to eccentric compressive load, complete the FBD to find M. Starting with

$EI\,v'' = M = \text{-------}$, derive secant formula $\sigma_{max} = -\frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right]$

4) 10. A rigid bar subjected to load P is supported by 2 columns. Set up an equation to find the location b of P so that the value P_{cr} for buckling is the largest possible and find this P_{cr}.



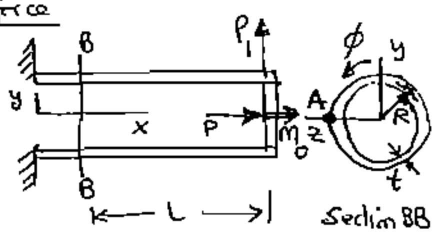
$P_{cr} =$

eq. for b:

For remaining problems do not simplify $+, -, \times, \div, \int(\,)dx$.

NOTE: For problems 11, 12, 13 include contribution of shear force

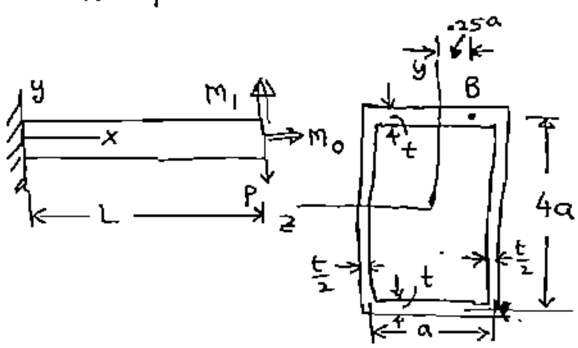
11. A thin tube of mean radius R, thickness t is closed at one end. It is subjected to internal pressure p, axial force P and axial moment M_0 and transverse load P_1. For point A on the cross-section BB:



(1,1+2) $\sigma_x =$ $\sigma_\phi =$ $\sigma_{x\phi} =$

(2) Express the 3 principal stresses in terms of $\sigma_x, \sigma_\phi, \sigma_{x\phi}$:

12. A cantilever thin tube is subjected to end moments M_0, M_1 and end force P. For point B in the fixed section: [$A = 6at$, $I_{zz} = 13.33\,a^3t$, $I_{yy} = 1.17\,a^3t$]

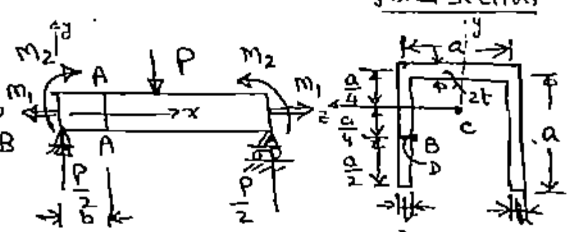


fixed section

(2+2) $\sigma_x =$

(2+2) $\sigma_{xz} =$

13. A simply-supported beam of channel section, with I_{yy}, I_{zz} at C, is subjected to the loads shown (the reactions are given). For points B and D on the cross-section AA



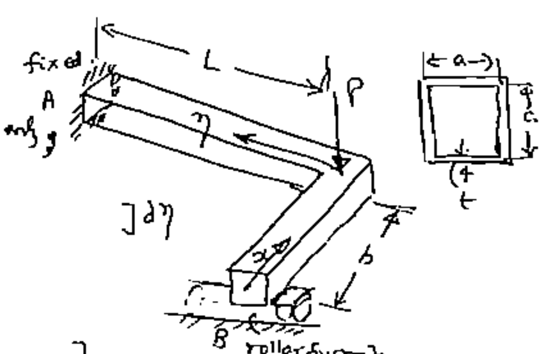
D is at middle of thickness and B is on inside

Section AA

(2, 2+2) $\sigma_x^B =$ $\sigma_{xy}^B =$

(2) $\sigma_{xy}^D =$

14. An L shaped square tube of thickness t is fixed at A and has roller support at B. Let R be the reaction at B. Including contribution of bending moment & twisting moment and stiffnesses EI and GJ:



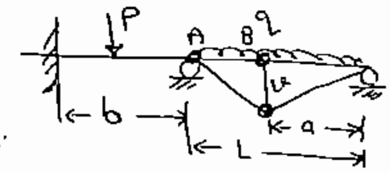
(1+2) $U = \int_0^b [\qquad\qquad] dx + \int_0^L [\qquad\qquad] d\eta$

(1) R is obtained using: . . . .

(2) For $t \ll a$: $EI = E[\qquad\qquad]$

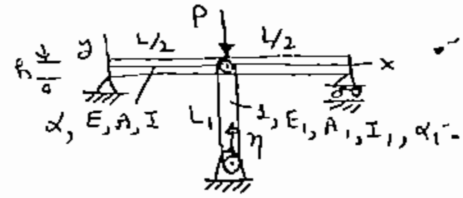
(4) $GJ = \frac{M_t}{\alpha} =$

(4) 15. The collapse mode of a cantilever propped at two points consists of two plastic hinges formed at A and B. Limit moment is M_L. Set up virtual work equation for finding q:



$\delta W =$

16. A beam is supported at the centre by bar 1. For beam temperature rise $\theta = \frac{1}{2}(T_1 + T_2) + (T_1 - T_2)\frac{y}{h}$ and for bar 1 temperature rise is uniform: $\theta = T_2$. Let R be



(1,1) the compressive force in bar 1. For bar 1: $N_{\theta_1} =$ $M_\theta =$

(1,1) for beam: $N_\theta =$ $M_\theta =$

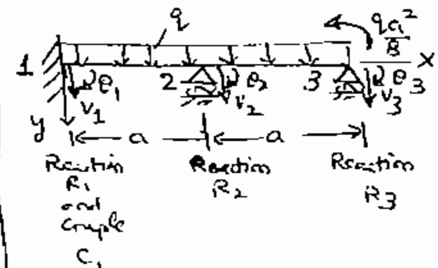
for the system

(4) $U^c = 2\int_0^{L/2} \Big[$ $\Big] dx$

(4) $+ \int_0^{L_1} \Big[$ $\Big] d\eta$.

(1) Which equation is used to find R:

17. Assemble six equations for beam (EI) shown.



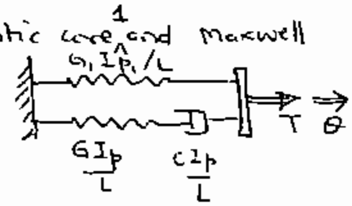
(3,4,3)

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} \; & \; & \; & \; & \; & \; \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{array}\right] \end{array} \begin{bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \\ V_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

List zero values if any for V_i, θ_i. Identify six unknowns in above equations

18. The mechanical model of a composite shaft in torsion with elastic core 1 and Maxwell annulus 2 is shown (a) Set up differential equation relating

(4) $T, \dot{T}, \theta, \dot{\theta}$. The radius of core is R_1 and outer radius of annulus is R_2.



(b) For step torque T, find $\theta(0)$, $\theta(\infty)$, $\sigma_{z\phi}^{max}(0)$, $\sigma_{z\phi}^{max}(\infty)$ for core 1 and annulus 2

(2) $t = 0$: $\theta =$

(2) $(\sigma_{z\phi}^{max})_1 =$ $(\sigma_{z\phi}^{max})_2 =$

(1) $t = \infty$: $\theta =$

(2) $(\sigma_{z\phi}^{max})_1 =$ $(\sigma_{z\phi}^{max})_2 =$

(2,1) (c) For step twist θ find $T(0)$, $T(\infty)$: