APL 104: Minor 1

Full Marks: 40   Duration: 1 hrs   Date: 29th Aug 2017   40

1. All questions are compulsory.
2. Problems 1-4 are objective type and has only one correct option each. You only need to provide the correct options to these questions on the first page of your answer sheet only. 10

Problem 1: Traction on how many planes are needed to completely define the stress tensor at a point? 10
(a) 1 (b) 2 (c) 3 (d)none of these (2)

Problem 2: The stress tensor at a point is such that its matrix representation remains the same in all coordinate system. What can you say about this stress tensor?
(a) The stress tensor is proportional to identity (b) The deviatoric part of stress tensor vanishes
(c) It is always the case in case of static fluids (d) all of these (3)

Problem 3: Suppose that two of the eigenvalues of stress tensor at a point becomes the same. How many principal planes will exist in such a situation?
(a) 3 (b) 2 (c) infinite (d) None of these (3)

Problem 4: The plane on which maximum shear traction acts
(a) has no normal traction (b) does not exist (c) has zero normal traction if two of principal stress components are equal and opposite (d) none of these (2)

Problem 5: Suppose the traction vector on three perpendicular planes with normals $(\underline{a}, \underline{b}, \underline{c})$ are as follows: 11

$$\begin{aligned} \underline{t}^a &= -1\ \underline{e}_1 + \sqrt{3}\ \underline{e}_2 \\ \underline{t}^b &= \sqrt{3}\ \underline{e}_1 + 1\ \underline{e}_2 \\ \underline{t}^c &= 10\ \underline{e}_3 \end{aligned}$$

Here $(\underline{a} = \frac{1}{2}(\sqrt{3}\underline{e}_1 + \underline{e}_2),\ \underline{b} = \frac{1}{2}(-\underline{e}_1 + \sqrt{3}\underline{e}_2),\ \underline{c} = \underline{e}_3)$ and $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ are the standard basis of cartesian coordinate system.
(a) Write down the corresponding stress tensor. (3)
(b) Write down the stress matrix in the coordinate system of both $(\underline{a}, \underline{b}, \underline{c})$ and $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$. (8)

Problem 6: The stress tensor at a point is given by the following matrix in cartesian coordinate system: 15

$$\underline{\underline{\sigma}} = \begin{bmatrix} -4 & 4 & 0 \\ 4 & -4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$\frac{3}{2}a - \frac{\sqrt{3}}{2}$

(a) Draw Mohr's circle corresponding to this state for traction on planes whose normal lie in (x-y) plane. (2) 15

$\sqrt{3} + 2 - 10\sqrt{3} = 2$

$\sqrt{2}a = \frac{\sqrt{3}}{2} e_1 + \frac{\sqrt{3}}{2} e_2$

$\sqrt{2}a = \frac{\sqrt{3}}{2} e_1 + \frac{1}{2} e_2$ $\quad \frac{1}{2} + \frac{3}{2}$

(b) What are the principal stress components and the corresponding principal planes? What is the maximum shear traction and on what plane does it act? (6)

(c) Using Mohr's circle idea, find out the normal and shear tractions on a plane whose normal lies in $x - y$ plane and makes an angle of 22.5 degrees from the x-axis in clockwise direction. (3)

(d) Find out the octahedral normal and shear tractions corresponding to this state of stress. (2)

(e) Decompose the given stress matrix into hydrostatic and deviatoric part. (2)

Problem 7: Suppose the traction on a plane with normal $\underline{a}$ is given by $\underline{t}^a$ while that on plane with normal $\underline{b}$ is given by $\underline{t}^b$. Furthermore, the two plane normals need not be perpendicular to each other. Prove that: (4) 4

$$\underline{t}^a \cdot \underline{b} = \underline{t}^b \cdot \underline{a}$$

$$\begin{bmatrix} -4 & 4 & 0 \\ 4 & -4 & 0 \\ 0 & 0 & 8 \end{bmatrix} \quad \begin{bmatrix} \checkmark & & \\ & \checkmark & \\ & & 8 \end{bmatrix}$$