22 September 2021

DEPARTMENT OF APPLIED MECHANICS
APL104: Mechanics of Solids
Second Semester 2021-22
Minor 1: Subjective **Maximum Marks 35**

- **Write your answer(s) on the answer sheets provided. Mobiles strictly prohibited.**
- **You may use a calculator.**
- **Start each problem on a new page and upload your answers on gradescope at the end of the exam.**

Problem 1:

Given the following displacement field in an isotropic linear elastic solid (E = 70 GPa and ν = 0.3):

$$u_1 = k_1 x_2 x_3, \qquad u_2 = k_2 x_1 x_3, \qquad u_3 = k_3 x_1 x_2$$

where k_1, k_2 and k_3 denote non-zero constants.

(a) In the absence of body forces, is the solid under equilibrium? (Hint: Calculate the stress tensor)
(b) Draw the Mohr's circle for stress at a point $(x_1, x_2, 1)$ for the special case of $k_1 = k_2 = -k_3 = 10^{-4}$.
(c) Using the Mohr's circle in part (b), calculate the principal stresses and maximum shear stress. Identify the corresponding principal and maximum shear planes. **[5, 5, 5]**

Problem 2:

Unit elongations (change in length per unit length), are measured experimentally in 3 directions that are 45^0 apart. The unit elongations along $\boldsymbol{e_1}$, $(\boldsymbol{e_1} + \boldsymbol{e_2})/\sqrt{2}$ and $\boldsymbol{e_2}$ are denoted by a, b and c, respectively.

(a) Obtain the strain components ϵ_{11}, ϵ_{22} and ϵ_{12}.

(b) Assuming plane strain, calculate the volumetric and deviatorial strain components. **[8]**

Problem 3:

A rectangular bar of constant cross-sectional area *A* is axially loaded at its one-third points and rigidly restrained at the ends (Fig. 1).

(a) Find the support reactions, if the bar was free from stress before the application of external load.

(b) Find the axial stress in each of the sections. **[12]**

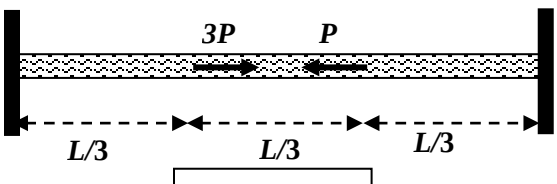

Figure 1

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Important Formulae:

Stress transformation:

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'y'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Infinitesimal strain: $\epsilon_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$

Stress-strain relationship (Isotropic): $\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E}$

Shear Modulus: $G = \frac{E}{2(1+\nu)}$

Axially loaded bar: $\delta = \frac{PL}{AE}$