APL 104: Minor 2

Full Marks: 35 Duration: 1 hrs Date: 4^{th} Oct 2016

Problem 1: Pressurizing hollow circular tubes: Often a hollow tube is pressurized both internally (p^{in}) and externally (p^{out}). Application of pressure generates stress within the tube. Assume that the tube's inner radius is R_1 and outer radius R_2. Further assume that the tube is not allowed to stretch/compress axially.

(a) Find out the formula for variation in σ_{rr} and $\sigma_{\theta\theta}$ as a function of the tube's radial coordinate. Also draw sketches/plots for their variation with respect to ''r'. (3+3)
(b) Find out the formula for radial displacement that generates in the tube. (3)
(c) Find out the expression for axial force that generates in the tube? (3)
(d) Suppose the tube were free to expand axially? How much would be the axial strain generated in that case? (3)

Problem 2: Think of a solid beam having square cross-section of side length h and axial length L. The beam's axis lies along the z axis while its cross-section's sides lie along x and y axes. Suppose the beam is stretched by applying axial force P to it such that its cross-section remains square and planar even after deformation. Also assume the deformation to be axially homogeneous. Let us think of using the Cartesian coordinate system.

(a) What coordinates (x,y,z) will the displacement functions (u_x, u_y, u_z) depend on? Give reasons for your answer. (3)
(b) Find out the strain matrix and the stress matrix in terms of displacement functions and the material parameters (λ, μ) in Cartesian coordinate system. (5)
(c) Substitute the expressions for stress components in the equilibrium equations of Cartesian coordiante system (assume no body force/acceleration) and obtain the equations. Write down the boundary conditions too. Solve them to obtain the three displacement functions. (7)

Problem 3: Show that in case of isotropic bodies, the stress tensor and the strain tensor will both have the same set of principal directions. Further show that the set of planes whose normals are parallel to one of the principal directions do not slide relative to each other. (5)