Department of Applied Mechanics
Major Test AML 150

02/05/2008

N.B. Assume missing data if any

Q1. If the state of stress at a point is

$$\tau_{ij} = \begin{pmatrix} 100 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & -100 \end{pmatrix} N/mm^2.$$

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Find (a) Magnitude of shear stress on a plane whose normal is in the direction of $i + j + k$, and

(b) Maximum shear stress

Q2. If stress tensor at a point is

$$\tau_{ij} = \begin{pmatrix} x^2+y & xy^2 & yz^2 \\ xy^2 & x+y^2 & x^2 z \\ yz^2 & x^2 z & z^2+x \end{pmatrix}$$

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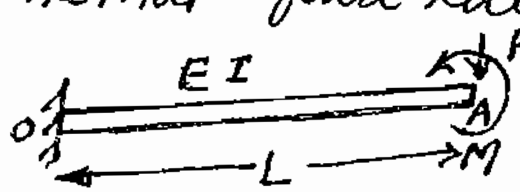
Find the body forces to maintain the equilibrium.

Q3. Using energy method find the deflection at a point A



cantelever beam

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Q4 Derive the Critical load for the column fixed at one end and hinged at the other

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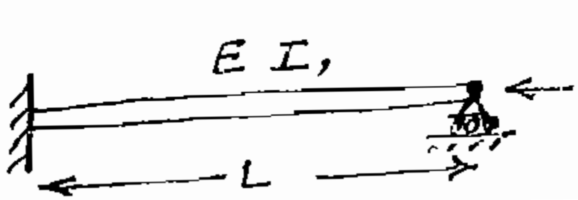
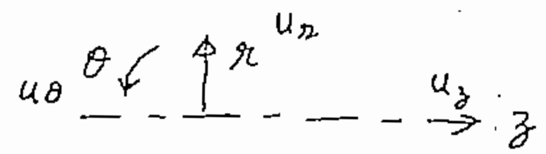


DEPT. OF APPLIED MECHANICS

FLUID MECHANICS



For confined flows, the body-force & pressure terms can be combined into piezometric pressure. $P = p + \rho g h$

N.S. Eqns.

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}$$
$$= B_r - \frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right\} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right]$$

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}$$
$$= B_\theta - \frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta} + \nu\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right\} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right]$$

$$\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}$$
$$= B_z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right]$$

Continuity Eqn. (Incompressible)

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0 \quad ; \text{ or, } \quad \frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0.$$

Stokes's relations.

$$\sigma_{rr} = -p + 2\mu\frac{\partial u_r}{\partial r} \; ; \; \sigma_{\theta\theta} = -p + 2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right) ; \; \sigma_{zz} = -p + 2\mu\frac{\partial u_z}{\partial z}$$

$$\tau_{r\theta} = \mu\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \mu\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z}\right) = \tau_{z\theta}$$