AML160 Mechanics of Fluids 11.00 am to 12.00 noon

(Max marks 48) 7 October 2013

Please attempt all questions. Marks for each question are given alongside.

Data: $\rho_w = 1000$ kg/m^3 $\rho_{air} = 1$ kg/m^3 $\mu_w = 10^{-3}$ Kg/(ms) $\mu_{air} = 1.5 \times 10^{-5}$ kg/(ms)

Q1) A cylindrical body travels through water at a constant speed $U = 20$m/s, as shown. At this instant at point B the induced velocity is $0.5U$ and at point C the velocity is $0.3U$ in the direction shown. You may assume that the fluid is inviscid. (12)

i) Determine $P_B - P_C$.

ii) If the distance between points B and C is L =0.5m, then determine the average rate of change of speed between B and C at this instant.

(Neglect changes in gravitational potential between B and C.)

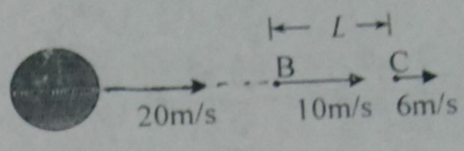

Figure Q1

Q2) A rectangular block of mass M, with vertical faces, rolls on a horizontal surface between two opposing jets as shown. The jets and the block are aligned so that there is no net torque on the block. At $t=0$, the block is set in motion at speed U_0 to the right. Neglect the mass of liquid that may be sticking to the block.

i) Obtain general expressions for the acceleration of the block $a(t)$ and its speed $U(t)$.

ii) Find the distance travelled by the block when the speed drops to $\frac{1}{e}U_0$. (12)

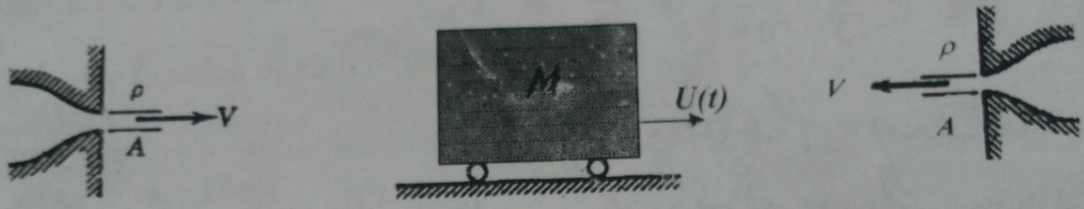

Figure Q2

Q3) Show that for 2-D incompressible, inviscid flows, the Euler equations, $\frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = -\frac{1}{\rho}\nabla P$

simply reduce to $\frac{D\vec{\Gamma}}{Dt} = 0$. Here $\vec{\Gamma}$, is the vorticity vector defined as $\vec{\Gamma} = \nabla \times \vec{V}$. (10)

Q4) Consider the flow field formed by a counter-clockwise vortex of strength K placed at the origin and a uniform flow, U, in the 'x' direction. Determine:

i) The combined velocity field, combined stream function and combined potential function.

ii) The stagnation point/points.

iii) The equation of the stagnation streamline.

iv) The pressure at $(x,y) = (0, \pm\frac{K}{2\pi U})$, given that the pressure far away from the origin is P_∞. (14)

---------o0o--------

Useful vector identities.

i) $\vec{V}\cdot\nabla\vec{V} = \nabla(\vec{V}\cdot\vec{V}/2) - \vec{V}\times(\nabla\times\vec{V})$

ii) $\nabla^2\vec{V} = \nabla(\nabla\cdot\vec{V}) - \nabla\times(\nabla\times\vec{V})$

iii) $\nabla\times(\nabla\phi) = 0.$ $\forall\phi.$

iv) $\nabla\cdot(\nabla\times\vec{\psi}) = 0.$ $\forall\vec{\psi}.$

v) $\nabla\times(\vec{V}\times\vec{\Gamma}) = \vec{\Gamma}\cdot\nabla\vec{V} - \vec{V}\cdot\nabla\vec{\Gamma}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$