Major Exam

Introduction to Continuum Mechanics, APL 701, 19/Nov/2018

Attempt all questions.

Q: Part I (1-15), 2 marks each, Q: Part II (1-4) 5 marks each

Part I

1. Prove that, $\frac{1}{J}\frac{DJ}{Dt} = div(\mathbf{v})$

2. Show that, under an objective transformation given by $x^* = c(t) + Q(t)(x - x^0)$, $C = F^T F$ right Cauchy-Green tensor is not objective and Left Cauchy-Green tensor $B = FF^T$ is an objective tensor.

3. Prove that material derivative of an objective tensor is not objective

$$\dot{T}^* = \dot{Q}TQ^T + Q\dot{T}Q^T + QT\dot{Q}^T$$

4. The 2nd PK stress $\tilde{T}$ is related to 1st PK stress tensor T_0 by the formula, $\tilde{T} = F^{-1}T_0$. Show that in an objective transformation, $\tilde{T}^* = \tilde{T}$

5. A velocity field is given by $v_1 = k\, x_2^2,\ v_2 = v_3 = 0$. Find the rate in which work is converted to heat energy, if μ is viscosity of the fluid.

6. $\varphi = x_1^3 - 3x_1x_2^2$, show φ satisfies Laplace equation, find the irrotational velocity field.

7. Prove that for an isotropic Hookean material, the principal directions of stress and strain coincide.

8. Given the displacement field, $u_1 = k\, X_2X_3,\ u_2 = k\, X_1X_3,\ u_3 = k\,(X_1X_2 + X_3^2)$ in the absence of body force, is the state of stress a possible equilibrium stress field?

9. A rotation tensor is given by $\boldsymbol{R}\mathbf{e}_1 = \mathbf{e}_2,\ \boldsymbol{R}\mathbf{e}_2 = \mathbf{e}_3,\ \boldsymbol{R}\mathbf{e}_3 = \mathbf{e}_1$, prove that the dual vector of the anti-symmetric part of $\boldsymbol{R}$ is $\boldsymbol{t}^A = \frac{1}{2}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)$.

10. If $\mathbf{n}$ is an eigen vector of $\boldsymbol{U}$ (right stretch tensor), and λ is the eigen value of of $\boldsymbol{U}$. Find the eigen vector and eigen value of $\boldsymbol{V}$.

11. Given, $\alpha = \alpha(\mathbf{r})$ is a scalar, and $\boldsymbol{t} = \boldsymbol{t}(\mathbf{r})$ a vector, prove that

$$div(\alpha \boldsymbol{t}) = \boldsymbol{t}(\nabla\alpha) + \alpha\ div(\boldsymbol{t})$$

12. The density of a continuum is given by $\rho = \rho(t)$ and the velocity field is given by $\mathbf{v} = \alpha(x_1 t\mathbf{e}_1 + x_2 t\mathbf{e}_2)$, the density at $t = t_0$ is given to be ρ_0, from mass balance equation find the current density of the continuum.

13. Let, **m** and **n** are two unit vectors that define two planes, M and N that passes through a point P. For an arbitrary state of stress defined at the point P, prove that $\mathbf{n} \cdot \mathbf{t}_m = \mathbf{m} \cdot \mathbf{t}_n$

14. Show that for an incompressible material ($\nu \to 1/2$) that (a) $\mu = EY/3$, $\lambda \to \infty$, $k \to \infty$, but $k - \lambda = {2\mu}/{3}$ a finite quantity.

15. Prove that, $\mathbf{v} \cdot div(\mathbf{T}) = div(\mathbf{T}\mathbf{v}) - tr(\mathbf{T}\mathbf{D})$

Part II

1. The state of stress at a point is

$$[T] = \begin{bmatrix} 300 & 0 & 0 \\ 0 & -200 & 0 \\ 0 & 0 & 400 \end{bmatrix}$$

Find the magnitude of the normal and shearing stress on the plane whose normal is in the direction of $(2\mathbf{e}_1 + 2\mathbf{e}_2 + \mathbf{e}_3)$.

2. For a continuum body, the constitutive equation for the incompressible material ($div(\mathbf{v}) = 0$) is given by $T = (-p + \lambda\,\mathrm{tr}(D))\,I + 2\mu D$, the Cauchy's first equation of motion is given by $\rho a = div(T) + \rho b$. If λ, μ are constants, Prove that,

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\nabla \mathbf{v})\mathbf{v}\right] = \rho b - \nabla p + \mu \nabla^2 \mathbf{v}$$

3. Given the velocity field of a linear viscous fluid given as

$$v_1 = k\,x_1,\ v_2 = -k\,x_2,\ v_3 = 0$$

show that,
a. The velocity field is irrotational
b. Find the acceleration field.
c. Find the stress tensor.
d. Neglect body force, take p=p0 at the origin, use the Bernoulli's equation to find the pressure distribution.
e. Is no-slip boundary condition satisfied at $x_2 = 0$

4. Show that for any function $f(s)$, the displacement $u_1 = f(s)$, where $s = x_1 + c_l t$ satisfies the wave equation.

$$\frac{\partial^2 u_1}{\partial t^2} = c_l^2 \frac{\partial^2 u_1}{\partial x_1^2}$$