APL 701

Minor- 1

28 August 2016

1. Convert the following equations to index notation:

(a) $\rho \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{v} = \rho \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v}$ (b) $\mathbf{E} = \frac{\nabla u + (\nabla u)^T}{2} + \frac{\nabla u \bullet (\nabla u)^T}{2}$ (5 marks)

2. Consider a homogeneous continuum undergoing a uniform temperature change given by $\Delta\theta = \theta - \theta_0$. It is known that the relation between the thermal strain tensor e_{ij} and $\Delta\theta$ is given by $e_{ij} = \alpha_{ij}\Delta\theta$, where α_{ij} is the thermal expansion coefficient tensor. We wish to see what properties the α_{ij} tensor must have if the material has planes of symmetry.

(a) Let S_i be the plane with normal e_i. Let Q_i be the reflection tensor that represents reflection across this plane S_i. Write a matrix representation of Q_i for i = 1,2,3. (4 marks)

(b) Suppose we change our coordinate axes from **e** to **e′** through a reflection across S_1. What is the set of axes **e′**? How would the *components* of the thermal expansion coefficient tensor change due to this change of coordinates? (10 marks)

(c) If S_1 is a plane of material symmetry, this means that the components of its expansion coefficient do not change due to reflection. Use the results of (b) to infer that there are therefore only 5 non-zero components of α_{ij} and mention which ones these are. (5 marks)

(d) Now, if S_2 and S_3 are also planes of material symmetry, how many non-zero components of α_{ij} are there when there are 3 planes of material symmetry? (6 marks)

3. For a given tensor **T** it is known that $p^T q = p^T \mathbf{T} q$ for all vectors p and q. Prove that this means that **T** must be the identity tensor. (5 marks)

4. Consider the displacement field: $u_1 = k(2X_1^2 + X_1X_2)$, $u_2 = kX_2^2$, $u_3 = 0$

(a) Two material elements $d\mathbf{X}^{(1)} = dX_1\mathbf{e}_1$ and $d\mathbf{X}^{(2)} = dX_2\mathbf{e}_2$ emanate from a particle designated by $\mathbf{X} = \mathbf{e}_1 + \mathbf{e}_2$. Find the new lengths and the new angles between these two elements after being subjected to the above displacement field. Do not assume small displacements. (15 marks)

(b) Substitute $k = 10^{-4}$ in the above and find out numbers for the above quantities. How would you have calculated the above quantities quickly knowing that the displacements are now small? Compare your previous answer using this alternate method. (10 marks)