Thursday, August 31, 2017

APL 701 – Continuum Mechanics

Minor 1

Problem 1: Given the deformation defined by **(20 points)**

$$x_1 = X_1, \qquad x_2 = X_2 + \frac{1}{2}X_3^2, \qquad x_3 = X_3.$$

a) Sketch the deformed shape of a unit square OABC in the plane $X_1 = 0$.

b) Determine the differential elements $d\boldsymbol{x}^{(2)}$ and $d\boldsymbol{x}^{(3)}$ resulting from the deformation of length elements $d\boldsymbol{X}^{(2)} = dS^{(2)}\boldsymbol{e}_2$ and $d\boldsymbol{X}^{(3)} = dS^{(3)}\boldsymbol{e}_3$ (originally at point C in reference configuration), respectively.

c) Determine the change in the original right angle between the elements $d\boldsymbol{x}^{(2)}$ and $d\boldsymbol{x}^{(3)}$.

d) Compute the stretch at B in the direction of the unit normal $N = \frac{1}{\sqrt{2}}(\boldsymbol{e}_2 + \boldsymbol{e}_3)$.

Problem 2: Prove the following using indicial notation **(20 points)**

a) Given $A_{ij} = \delta_{ij}B_{kk} + 3B_{ij}$, prove $B_{ij} = \frac{1}{3}A_{ij} - \frac{1}{18}\delta_{ij}A_{kk}$. (Hint: first obtain the expression for B_{kk} in terms of A_{jj}).

b) Given a skew-symmetric tensor B_{jk} and a vector v_i defined by $v_i = \varepsilon_{ijk}B_{jk}$, prove that $B_{mq} = \frac{1}{2}\varepsilon_{mqi}v_i$. Use the identity $\varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$.

Problem 3: Rate of deformation, velocity gradient etc.

3.1. Consider an element in spatial configuration $d\boldsymbol{x} = ds\,\boldsymbol{n}$. **(10 points)**

Show that $\dot{\boldsymbol{n}} = \boldsymbol{D}\boldsymbol{n} + \boldsymbol{W}\boldsymbol{n} - (\boldsymbol{n}.\boldsymbol{D}\boldsymbol{n})\boldsymbol{n}$, where $\boldsymbol{D}$ and $\boldsymbol{W}$ correspond to rate of deformation and vorticity tensor, respectively.

3.2. For the deformation map **(15 points)**

$$x_1 = X_1, \qquad x_2 = X_2e^t + X_1(e^t - 1), \qquad x_3 = X_3 + X_1(e^t - e^{-t}).$$

a) Obtain the velocity field $v_i(\boldsymbol{x})$, gradient of velocity $\boldsymbol{L}$, and deformation gradient tensor $\boldsymbol{F}$.

b) Verify the relationship $\boldsymbol{L} = \dot{\boldsymbol{F}} \cdot \boldsymbol{F}^{-1}$ for this motion.

$F L = \dot{F}$

any one

Problem 4: Polar decomposition and eigenvalue analysis **(15 points)**

a) Show that the eigenvalues of **U** and **V** are the same and obtain the relationship between their eigenvectors.

b) Given two *distinct eigenvalues* $\lambda_1 \& \lambda_2$ of a symmetric second-order tensor **T** show that their corresponding eigenvectors $\boldsymbol{n}_1 \& \boldsymbol{n}_2$ are orthogonal to each other.