

Department of Applied Mechanics, Indian Institute of Technology Delhi

First Semester, 2023-2024

APL 701 Continuum Mechanics
Mid-semester Examination (Open Book)

Prof Gaurav Singh

Date: 14/09/2023

Duration: 120 min

Maximum Marks: 60

Instructions: a. All questions are compulsory. b. Clearly state the rationale behind each step. c. Only hand-written notes are allowed.

Problem 1

10 marks

With respect to superposed material axes X_i and the spatial axes x_i , the displacement field of a continuum body is given by $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$ where A is a constant. Determine the displacement vector components in both material and spatial forms.

Problem 2

10 marks

Prove that $\sigma_{ij}\sigma_{ik}\sigma_{kj}$ is an invariant of the stress tensor.

Problem 3

10 marks

Show that the sum of squares of the magnitudes of the stress vectors on the co-ordinate planes is independent of the orientation of the coordinate axes.

Problem 4

30 marks

The state of stress throughout a body is given by the stress tensor

$$\sigma_{ij} = \begin{pmatrix} 0 & Cx_3 & 0 \\ Cx_3 & 0 & -Cx_1 \\ 0 & -Cx_1 & 0 \end{pmatrix}$$

where C is an arbitrary constant. (a) Show that the equilibrium equations are satisfied if body forces are zero [05 marks]. (b) At the point $P(4,-4,7)$ calculate the stress vector on the plane $2x_1 + 2x_2 - x_3 = -7$ [05 marks], and on the sphere $x_1^2 + x_2^2 + x_3^2 = 9^2$ [05 marks]. (c) Determine the principal stresses, maximum shear stresses and principal deviator stresses at P [05 marks]. (d) Sketch the Mohr's circles for the state of stress at P [10 marks].