October 7, 2017

APL 701 Continuum Mechanics – Minor 2

Maximum marks = 60 (all questions carry equal weightage)

Problem 1: Prove the energy theorem using the mass, momentum and angular momentum conservation statements:

$$\int_{\mathcal{P}} \mathbf{b} \cdot \mathbf{v} \rho \, dv + \int_{\partial \mathcal{P}} \mathbf{t} \cdot \mathbf{v} \, da - \frac{d}{dt} \int_{\mathcal{P}} \frac{1}{2} \mathbf{v} \cdot \mathbf{v} \rho \, dv = \int_{\mathcal{P}} \mathbf{T} \cdot \mathbf{D} \, dv.$$

Problem 2: For a body under superposed rigid body motion ($\boldsymbol{x}^+ = \boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c},\ t^+ = t + a$), prove that

$$\boldsymbol{v}^+ = \boldsymbol{\Omega}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{Q}\boldsymbol{v} + \dot{\boldsymbol{c}}, \quad \boldsymbol{L}^+ = \boldsymbol{Q}\boldsymbol{L}\boldsymbol{Q}^T + \boldsymbol{\Omega} \quad where \quad \boldsymbol{\Omega} = \dot{\boldsymbol{Q}}\boldsymbol{Q}^T$$

Also, obtain the expressions for vorticity tensor $\boldsymbol{W}^+$ and rate of deformation tensor $\boldsymbol{D}^+$. Which of these quantities are objective?

Problem 3: Given the displacement field in an isotropic linear elastic solid

$$u_1 = kX_2X_3, \quad u_2 = kX_1X_3, \quad u_3 = k(X_1^2 - X_2^2), \quad where\ k = 10^{-4}$$

(a) Find the strain and stress components (in terms of Lame's constants).

(b) What is the required body force for the solid to be in static equilibrium?

(Note: Solve only one of Problems 4 and 5)

Problem 4: Show that the strain energy U can be split into dilatation energy U_1 and distortion energy U_2, respectively, i.e.,

$$U = U_1 + U_2 = \frac{1}{6} T_{ii}\varepsilon_{jj} + \frac{1}{2} T_{ij}^d \varepsilon_{ij}^d$$

where T_{ij}^d and ε_{ij}^d represent the deviatoric part of stress and strain tensors defined by:

$$T_{ij}^d = T_{ij} - \frac{1}{3} T_{kk}\delta_{ij} \quad and \quad \varepsilon_{ij}^d = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}$$

Problem 5: Let a material have the constitutive equation

$$T_{ij} = \alpha\delta_{ij}D_{kk} + 2\beta D_{ij}$$

where α and β are material constants, T_{ij} are the components of the stress tensor and D_{ij} are the components of the rate of deformation tensor (i.e., the symmetric part of $\boldsymbol{L}$). Show that the equation of motion of the material in terms of velocity gradient is reduced to

$$\rho\dot{v}_i = \rho b_i + (\alpha + \beta)v_{j,ij} + \beta v_{i,jj}$$