APL 701, **Introduction to Continuum Mechanics**, **Minor II**, October 5th, 2018

Attempt all questions

(Q1: 5 marks, Q2-6: 4 marks each, Q7: 5 marks, Q8: 10 marks)

1. Prove $\mathbf{n}dA = J\,\mathbf{F}^{-T}\mathbf{n}_0 dA_0$
2. A tensor transforms every vector into its image with respect to the plane whose normal is $\mathbf{e}_2$. $\mathbf{e}_i$ are the basis vectors. Find the matrix of $\mathbf{T}$. (Hint $\mathbf{T}\mathbf{e}_2 = -\mathbf{e}_2$)
3. Prove that only possible real eigen-values of an orthogonal tensor $\mathbf{Q}$ are $\lambda = \pm 1$
4. Prove that $\dot{\mathbf{R}}\mathbf{R}^T$ is an anti-symmetric tensor. $\mathbf{R}$ is a rotation tensor.
5. Prove that $\nabla\mathbf{v} = \dot{\mathbf{F}}\mathbf{F}^{-1}$.
6. Prove for volume preserving continuum deformation (isochoric) $\text{div}(\mathbf{v}) = 0$
7. Consider the motion given by

   $\mathbf{x} = \mathbf{X} + X_1 k\mathbf{e}_1$, let $d\mathbf{X}_1 = \frac{dS_1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2)$, $d\mathbf{X}_2 = \frac{dS_2}{\sqrt{2}}(-\mathbf{e}_1 + \mathbf{e}_2)$

   a. Find the deformed elements $d\mathbf{x}_1$ & $d\mathbf{x}_2$

   b. Evaluate the stretches of these elements $\frac{ds_1}{dS_1}$ & $\frac{ds_2}{dS_2}$.
8. Given the following deformation in rectangular Cartesian coordinates $x_1 = 2X_2,\ x_2 = 3X_3,\ x_3 = X_1$ Find i) right Cauchy-Green tensor **C,** ii) right stretch tensor **U** iii) the rotation tensor **R** iv) The ratio of deformed volume to initial volume v) The Lagrangian strain tensor $\mathbf{E}^*$