APL703 Engineering Mathematics and Computation

Major Test

Duration: 90 minutes (9:30 am - 11:00 am) Date: 07/01/2021

Total Marks: 30

Instructions:

(a) Please write all answers clearly.
(b) Please do not copy someone's answer. (If found cheating/copying, marks will be zero.)
(c) Please read the question paper carefully.

Problem 1: (i) A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $T(X) = AX$, where $A = \begin{pmatrix} 1 & 2 \\ 4 & 8 \end{pmatrix}$ and $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Find $Ker\ T$ or null space of T and evaluate the nullity of T. (2+1)

(iii) Consider a 2×2 matrix A with $tr(A) = 2\delta$ and $det(A) = \delta^2$, where δ is a real number. Find eigenvalues of the matrix A and evaluate the algebraic multiplicity. (1+1)

Problem 2: Suppose $f(x)$ is bounded, integrable, periodic and continuous function over the interval $[-\pi\ \pi]$. Prove that

$$\|f(x)\|^2 = 2a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2),$$

where a_0, a_n and b_n are Fourier coefficients, and the inner product between two functions f and g is defined as $\langle f, g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)dx$. (5)

Problem 3: (i) Solve the following Darcy-Brinkman ODE

$$\frac{\mu}{\epsilon}\frac{d^2u}{dy^2} - \frac{\mu}{\kappa}u = \frac{dp}{dx},$$

where μ, κ and ϵ are real constants. Further, $u = u(y)$ is a function of y alone while $p = p(x)$ is a function of x alone. (5)

(ii) Suppose $y_1(x)$ and $y_2(x)$ are two independent solutions of the following ODE

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0.$$

Prove that $W(y_1, y_2) = W_0\, e^{-\int_{x_0}^{x}\frac{a_1(x)}{a_0(x)}dx}$, where $W(y_1, y_2)$ specifies the Wronskian of y_1 and y_2 and W_0 is the value of $W(y_1, y_2)$ when $x = x_0$. (5)

Problem 4: (i) Classify the following PDE

$$\nabla^2 u = \partial_{xx}u + \partial_{yy}u = 0$$

and determine its canonical form. (1+4)

(ii) Using the method of separation of variable solve the heat conduction equation

$$2\partial_{xx}u = \partial_t u, \qquad 0 \le x \le 3, \quad t \ge 0$$

with initial and boundary conditions $u(x, 0) = 5\sin 4\pi x$, $u(0, t) = 0$ and $u(3, t) = 0$. [You do not need to express the solution in series form at the end] (5)