APL703: Engineering Mathematics and Computation

Minor-I Test

Duration: 40 minutes | Date: 08/11/2020
Total Marks: 20

Instructions:

(a) Please write all answers clearly.
(b) Please do not copy someone's answer. (If found cheating/copying, marks will be zero.)
(c) Please read the question paper carefully.

Problem 1: (i) Suppose that a tensor Q corresponds to a right-hand rotation about x_3 axis by an angle θ ($\neq 0$). Find the tensor Q and check its orthogonality. Do the column vectors of Q form a linearly independent set in $\mathbb{R}^3$? (1+2+1)

(ii) Find the matrix corresponding to a tensor T which transforms a vector $\vec{a} = \hat{e}_1 + 2\hat{e}_2 + 3\hat{e}_3$ into another vector $\vec{b}$ by the following relation

$$\vec{b} = T\vec{a} = (\vec{m} \otimes \vec{n})\vec{a},$$

where $\otimes$ indicates the dyadic product and $\vec{m} = \frac{\sqrt{2}}{2}(\hat{e}_1 + \hat{e}_2)$, $\vec{n} = \frac{\sqrt{2}}{2}(-\hat{e}_1 + \hat{e}_3)$. (3)

(iii) Given that a tensor T satisfies $T_{ij} = -T_{ji}$ for all i, j in 3D space. Prove that $T_{ij}a_i a_j = 0$, where $\vec{a} = a_i\hat{e}_i$ relative to the orthogonal unit vectors $\{\hat{e}_i\}$. (3)

Problem 2: (i) Determine the value of c for which the column vectors $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ c \end{pmatrix}$ do not form a basis of $\mathbb{R}^3$. (3)

(ii) Find a basis for the subspace S of a given vector space $V \equiv \mathbb{R}^5$, where S is defined as

$$S = \left\{ \begin{bmatrix} a+b \\ b \\ c \\ 0 \\ c+b \end{bmatrix} : a,\ b,\ c \in \mathbb{R} \right\}.$$

What is the dimension of S or $dim\ S$? (2+1)

(iii) If u, v and w are linearly independent vectors in $\mathbb{R}^n$. For which values of $k \in \mathbb{R}$, the vectors $ku+v$, $v+kw$ and $w+ku$ are also linearly independent? (4)