Major Exam

ASL707

II Semester 2009-2010

Time: Two hours **Max Marks: 30**

You are allowed to bring with you one A4 size sheet of paper containing any statistical formulae etc. You are also required to bring the tables of Standard Gaussian and Gamma Distributions

1. Find the function minimizing the functional:

$$J[y(x)] = \frac{1}{2}\int_{-1}^{1}(-y'^2 + 2xy)dx$$

$$with\ y(-1) = 0; y(1) = 0$$ **(4 marks)**

2. In a land-surface model, the temperature in the soil layer is governed by:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2}, \qquad 0 < z < 1; \quad t > 0$$

with the conditions

$$T(z,0) = z,$$

$$T_z(0,t) = 0 \quad and \quad T_z(1,t) = 0.$$

(a) Find the analytical solution. **(5 marks)**

(b) Solve the equation numerically using explicit finite difference scheme and compute the values using your calculator at first time step. **(6 marks)**

3. Consider the bivariate normal distribution as a model for the Canandaigua maximum (x) and Canandaigua minimum (y) temperature data in TABLE – I.

(a) Fit the distribution parameters **(2 marks)**

(b) Using the fitted distribution, compute the probability that the maximum temperature will be as cold or colder than 20°F, given that the minimum temperature is 0°F. **(4 marks)**

4. The results of a linear regression analysis were produced as an ANOVA table. Unfortunately, the printer did not print some values on the paper clearly and what remains visible is reproduced below. The table entries that were not clear are marked **A**, **B** and **C**.

Source	**df**	**SS**	**MS**	**F**
Total	**A**	2327.419		
Regression	1	1985.798	1985.798	**C**
Residual	**B**	341.622	11.780	

From what you know about linear regression, find the following:

(a) The value of **A** **(1 mark)**

(b) The value of **B** **(1 mark)**

(c) The value of **C** **(1 mark)**

(d) The sample variance of the predictand. **(1 mark)**

(e) R^2 **(1 mark)**

5. For a given set of wind speed measurements of sample size 250, the probability of wind speeds exceeding a threshold value of 2 m/s is estimated to be 0.6. A theoretical model developed at IIT Delhi predicts that 208 of the 250 wind speeds would be greater than 2 m/s. You have been asked to make a judgment whether the data supports the theory. Is the data compatible with theory at a 5% significance level? Why? **(4 marks)**

----------END----------

TABLE - I

	Canandaigua Maximum Temperatures			Canandaigua Minimum Temperatures			
Date	x(i)	x(i)-xavg	(x(i)-xavg)**2	y(i)	y(i)-yavg	(y(i)-yavg)**2	(x(i)-xavg)* (y(i)-yavg)
1	34	2.23	4.97	28	7.77	60.37	17.33
2	36	4.23	17.89	28	7.77	60.37	32.87
3	30	-1.77	3.13	26	5.77	33.29	-10.21
4	29	-2.77	7.67	19	-1.23	1.51	3.41
5	30	-1.77	3.13	16	-4.23	17.89	7.49
6	35	3.23	10.43	24	3.77	14.21	12.18
7	44	12.23	149.57	26	5.77	33.29	70.57
8	38	6.23	38.81	24	3.77	14.21	23.49
9	31	-0.77	0.59	24	3.77	14.21	-2.90
10	33	1.23	1.51	29	8.77	76.91	10.79
11	39	7.23	52.27	29	8.77	76.91	63.41
12	33	1.23	1.51	27	6.77	45.83	8.33
13	34	2.23	4.97	31	10.77	115.99	24.02
14	39	7.23	52.27	26	5.77	33.29	41.72
15	51	19.23	369.79	38	17.77	315.77	341.72
16	44	12.23	149.57	23	2.77	7.67	33.88
17	25	-6.77	45.83	13	-7.23	52.27	48.95
18	34	2.23	4.97	14	-6.23	38.81	-13.89
19	36	4.23	17.89	28	7.77	60.37	32.87
20	29	-2.77	7.67	19	-1.23	1.51	3.41
21	27	-4.77	22.75	19	-1.23	1.51	5.87
22	29	-2.77	7.67	17	-3.23	10.43	8.95
23	27	-4.77	22.75	22	1.77	3.13	-8.44
24	24	-7.77	60.37	2	-18.23	332.33	141.65
25	11	-20.77	431.39	4	-16.23	263.41	337.10
26	21	-10.77	115.99	5	-15.23	231.95	164.03
27	19	-12.77	163.07	7	-13.23	175.03	168.95
28	26	-5.77	33.29	8	-12.23	149.57	70.57
29	28	-3.77	14.21	14	-6.23	38.81	23.49
30	31	-0.77	0.59	14	-6.23	38.81	4.80
31	38	6.23	38.81	23	2.77	7.67	17.26
Sum	**985.00**	***0.13***	**1855.42**	**627.00**	***-0.13***	**2327.4**	**1683.58**