DEPARTMENT OF CIVIL ENGINEERING: IIT DELHI

II SEMESTER 2015-2016 **DESIGN OF INDUSTRIAL STRUCTURES** **CEL 432**

MINOR II **Max. Marks: 20** **Time Limit: 1 hour**

Course Coordinator: Dr. Alok Madan

1. A circular cylindrical water tank consists of a reinforced concrete cylindrical shell structure with a mean radius 'a', shell thickness 't' and a height L. The water tank has **a light roof cover at the top (free end conditions) and a thick raft slab at the base (fixed end conditions)** constructed monolithically with the cylindrical shell. The edge loads at a fixed end are shown in Figure 1. Analytically, obtain the complete solution of the radial displacement 'w', meridional moment 'M_x', circumferential moment 'M_θ', radial shear force Q_x and tangential normal force 'N_θ' under the hydrostatic pressure due to water filled to the top, **considering the self-weight of the shell (i.e., self-weight of the shell has to be considered),** derive the complete analytical solutions of above variables in a closed form (in terms of x, a, t, H, γ_s, γ_w, E, λ and ν). The unit weights of the shell material and water are γ_s **and** γ_w, respectively. Some useful formulae are given below. (14 marks)

For a mean radius a = 2.5 m, shell thickness t = 250 mm and height L = 11 m, determine the numerical values of the meridional moment 'M_x', and tangential normal (hoop) force 'N_θ' at the **base and mid-height of the shell** from analytical solutions obtained above. Assume M25 concrete with unit weight γ_s = 25 kN/m^3, ν = 0.2 and elastic modulus $E_c = 5000\sqrt{f_{ck}}$ (MPa). (6 marks)

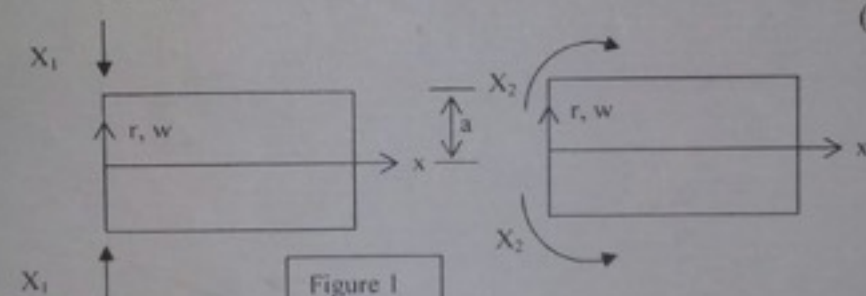

Figure 1

SOME USEFUL FORMULAE:

Bending Solution in a cylindrical shell due to edge force X_1 and edge moment X_2 (Fig. 1)

$$M_x = \frac{-X_1 a}{\lambda}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}\right) \qquad M_x = \sqrt{2}X_2\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}-\frac{\pi}{4}\right)$$

$$Q_x = -\sqrt{2}X_1\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}+\frac{\pi}{4}\right) \qquad Q_x = -\frac{\sqrt{2}X_2\lambda}{a}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}\right)$$

$$N_\theta = -2X_1\lambda\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}\right) \qquad N_\theta = \frac{-2\sqrt{2}X_2\lambda^2}{a}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}-\frac{\pi}{4}\right)$$

$$M_\theta = \nu M_x \qquad M_\theta = \nu M_x$$

$$w = -\frac{X_1 a^3}{2K\lambda^3}\exp\left(\frac{-\lambda x}{a}\right)\cos\left(\frac{\lambda x}{a}\right) \qquad w = \frac{-X_2 a^2}{\sqrt{2}K\lambda^2}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}-\frac{\pi}{4}\right)$$

$$\frac{dw}{dx} = \frac{X_1 a^2}{\sqrt{2}K\lambda^2}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}+\frac{\pi}{4}\right) \qquad \frac{dw}{dx} = \frac{X_2 a}{K\lambda}\exp\left(\frac{-\lambda x}{a}\right)\sin\left(\frac{\lambda x}{a}-\frac{\pi}{2}\right)$$

where, $K = \frac{Et^3}{12(1-\nu^2)}$; $\lambda^4 = 3(1-\nu^2)\frac{a^2}{t^2}$