Minor I: CLL113 Numerical Methods in Chemical Engineering

Total Marks: 15

Date: 28/08/16

1. Given the system of equations:

$$2x_1 + 3x_2 = 1,$$
$$4x_3 + ax_1 = b,$$
$$2x_2 + 6x_3 + cx_1 = 4,$$
$$2x_1 + 4x_2 + x_3 = 2.$$

(i) State the solvability and uniqueness conditions for this system in terms of unknowns a, b and c. (ii) For $a = 2$, $b = 8$ and $c = 1$, obtain the solution(s), if possible, using the Gauss elimination technique. Also, find the determinant of the coefficient matrix of the above system of equations. [1+2+1]

2. Is it possible to decompose following A matrix as A = LU? If yes, find matrices L and U using Doolittle's method: [1]

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

3. For the nonlinear algebraic equation $x^4 - x - 10 = 0$, determine the initial approximations for finding the smallest positive root. Use these to find the root correct to two decimal places using the Regula-Falsi method. [3]

4. Show that the power method converges to λ_2, the second largest eigenvalue, if the initial choice of vector $\mathbf{x}^{(0)}$ is selected to be orthogonal to $\mathbf{u}_1$, the eigenvector corresponding to the largest eigenvalue λ_1. Assume matrix A to be real symmetric matrix for which all eigenvectors form an orthogonal set of vectors. [2.5]

5. Consider the system of linear equations given below:

$$3x_1 - 10x_2 = 3$$
$$x_1 + x_2 = 2$$

Starting with the initial guess $x_1^{(0)} = 1$ and $x_2^{(0)} = 0$, obtain the solution correct up to three decimal places using the Gauss-Seidel method (without SOR). [2.5]

6. Consider the equation $e^x = 1 + x + (x^2/2) + (x^3/6)e^{0.3x}$. Using Newton-Raphson method, find the nontrivial positive root of the equation correct to four decimal places. [2]