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**CML100** **Minor 1** **Aug. 31, 2017** **Time: 60 min**

Consider a system with Hamiltonian $\hat{\mathcal{H}}$ and satisfying $\hat{\mathcal{H}}\psi_n = n^2\mathcal{E}_0\psi_n$. Consider, also, an observable $A$ represented by the operator $\hat{A}$. Action of $\hat{A}$ on $\psi_n$ follows $\hat{A}\psi_n = (n+1)a_0\psi_{n+1}$.

1. If the system is in the state $\psi_3$, what would be the result of (i) first measuring $\hat{\mathcal{H}}$ and then $\hat{A}$, and (ii) first measuring $\hat{A}$ and then $\hat{\mathcal{H}}$? 4
2. Comment on the significance of your result in question 1. 4

The radial Schrödinger equation for a particle of mass $M$ in a potential $V(r) = 1/2M\omega^2r^2$ is

$$-\frac{\hbar^2}{2M}\frac{d^2}{dr^2}u_{nl}(r) + \left[\frac{1}{2}M\omega^2r^2 + \frac{l(l+1)\hbar^2}{2Mr^2}\right]u_{nl}(r) = Eu_{nl}(r),$$

with $u_{nl}(r)$ defined as $u_{nl}(r) = rR_{nl}(r)$. Other symbols have their usual meaning. Here we are interested in this equation in the limit $r \to 0$.

3. What is the differential equation satisfied by $u_{nl}(r)$ in this limit? 3
4. Two possible solutions for $u_{nl}(r)$ are $r^{l+1}$ and $r^{-l}$. Which of these two solutions are acceptable as quantum mechanical wave functions? Why? 3

A hydrogen atom is in a state with a radial wave function $\frac{1}{\sqrt{3}}\left(\frac{1}{2a_0}\right)^{3/2}\left(\frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$.

5. In this state what fraction of the energy is in the form of potential energy? You are given that $\int_0^\infty r^n e^{-\beta r}\, dr = \frac{n!}{\beta^{n+1}}$. 5
6. Could an electron with this radial wave function have the angular wave function $\sqrt{\left(\frac{15}{8\pi}\right)}\cos\theta\sin\theta\exp(i\phi)$? Why or why not? 4

In the next two questions we explore the effect of a magnetic field on the energy levels in hydrogen atom, whose Hamiltonian in the absence of the magnetic field is $\hat{\mathcal{H}}_0$ and eigen functions are $\psi_{nlm}$. Classically, the motion of an electron around a closed loop produces a magnetic dipole given by

$$\vec{m} = -\frac{|e|}{2}\vec{r}\times\vec{v}$$

where $e$ and $\vec{v}$ are, respectively, the charge and the velocity of the electron. The potential energy of a magnetic dipole interacting with a magnetic field, $\vec{B}$, is given by

$$V = -\vec{m}\cdot\vec{B}.$$

Take the strength of the field to be $B_0$ and it is in the z-direction.

7. What is the Hamiltonian of the hydrogen atom in the magnetic field? 4
8. What is the effect of the magnetic field on the energy levels of a hydrogen atom in the 3$d$ state? Remember that $\hat{\mathcal{H}}_0\psi_{nlm} = E_n\psi_{nlm}$. 3