Date: 9/2/2014 CYL110 MINOR I Time: 1 hour Total Marks: 100

$\int_{-\infty}^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$; $\int_0^\infty e^{-ax^2} dx = \sqrt{\frac{\pi}{4a}}$; $\int_0^\infty x^n e^{-ax} dx = n!/a^{n+1}$; $h = 6.626\times10^{-34}$ Js, $c = 2.998\times10^{8}$ ms^{-1}, $m_e = 9.109\times10^{-31}$ kg, 1 amu $= 1.661\times10^{-27}$ kg

ATTEMPT QUESTIONS IN A SEQUENCE STARTING FROM Q1.

Q1. (a) A particle is in a state described by the wavefunction $\psi(x) = \left(\frac{2a}{\pi}\right)^{1/4} e^{-ax^2}$, where a is a constant and $-\infty \le x \le \infty$. Determine $\Delta x \Delta p_x$ by separately assessing uncertainty in momentum (i.e., $\langle p_x \rangle$ and $\langle p_x^2 \rangle$) and uncertainty in position (i.e., $\langle x \rangle$ and $\langle x^2 \rangle$). **(4 each = 16)**

(b) A particle is in a state described by the wavefunction $\psi(x) = a^{1/2} e^{-ax}$, where a is a constant and $0 \le x \le \infty$. Determine the expectation value of the commutator of the position and momentum operators. (9)

Q2. Consider a free particle of mass m in a 1-D infinite potential well of length L ($0 \le x \le L$). Now consider a wavefunction of the form

$$\psi(x) = Nx(L-x);\ 0 \le x \le L$$

(a) Is this an acceptable wavefunction for this system? Explain your answer. (5)

(b) What is N? (5)

(c) Find $\langle E \rangle$ for the system. (10)

(d) 'The uncertainty in energy, ΔE, for this system is <u>zero</u>'. Is this statement correct? Explain your answer. (*No calculation is to be done for this part*). (5)

Q3. (a) A cubic box of edge-length 1.2 nm contains 10 electrons. Applying the simple particle in a box theory, calculate the transition energy for the first excited-state of this system. Report your final answer in terms of wavelength in nanometer (nm). (15)

(b) Consider a function $u(r) = r\exp(-r)$ and an operator $\hat{O} = \left(\frac{d^2}{dr^2} + \frac{2}{r}\right)$. Is $u(r)$ an eigenfunction of $\hat{O}$? If yes, what is the eigenvalue, and if not, why not? Would $u(r)(0 \le r \le \infty)$ be an acceptable wavefunction for a quantum mechanical system? Justify. $(5 + 5 = 10)$

Q4. Consider a particle of mass m moving in the potential energy whose mathematical form is $V(x) = 0$ for $x < 0$ (region I) and $V(x) = V_0$ for $x > 0$ (region II), where V_0 is a constant.

(a) For $E > V_0$, write complete solutions to the Schrödinger equations in the two regions defining all parameters. Considering a particle traveling to the right in region I and excluding the case that the particle is traveling to the left in region II, what are appropriate wavefunctions now? $(3 + 3 + 2 + 2 = 10)$

(b) Write the complete wavefunctions in the two regions when $E < V_0$. $(3 + 3 = 6)$

(c) Lets say the wavefunction inside such a long barrier of height V_0 is $\psi = Ne^{-\kappa x}$. Calculate the probability that the particle is inside the barrier and the average penetration depth of the particle into the barrier. Is N a normalization constant? $(3 + 3 + 3 = 9)$