

Name: \_\_\_\_\_

Entry No: \_\_\_\_\_

CSL201: Data Structures. II semester, 2007-08.

Major Exam

3:30 PM to 5:30 PM, 3rd April 2008

Question	1	2	3	4	5	6	Total
Marks							

Note. Explain your solutions in words. No pseudocode. Write concisely and clearly. If necessary solve the problems on the rough sheet first and then copy clearly onto this sheet.

Q1 We consider a *deterministic* version of skip lists in this question (i.e. without the use of probability) called *1-2-3 skip lists*. Before we describe this structure, recall that each node in a skip list has a *height* associated with it (denoted  $h(\cdot)$ ) which corresponds to the highest list it belongs to i.e. if a node is only in the base list it has height 0, if it was promoted to the next level but not further up it has height 1 and so forth. Now, we describe 1-2-3 skip lists as the skip list dictionary structure on a set  $S$  which maintains the following invariant:

Given 2 keys  $x \leq y \in S$ , if  $h = \min\{h(x), h(y)\} \geq 1$  there must be at least one and at most 3 keys  $z \in S$  such that  $x < z < y$  and  $h(z) = h - 1$ . Sentinel nodes (at the two extremes) have the height of *current.max* + 1

Q1.1. Starting from an empty 1-2-3 skip list, insert the following items (in the order given): 3, 1, 6, 2, 9, 8, 4, 5, 14, 10, 11, 7, 12, 13, 15. You do not have to show all steps, just show the final skip list. (6 marks)

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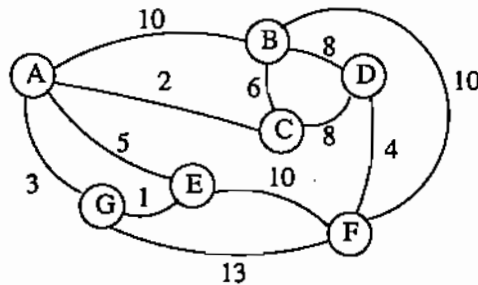
Q1.2. Give the algorithm for inserting an item into a 1-2-3 skip list as used in Q1.1. (5 marks)

Q1.3. Is the order of insertion important? Do you get the same 1-2-3 skip list no matter which order the keys are inserted in? If yes, prove your answer. If no, give a counter example. (5 marks)

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Q1.4. Show that the maximum height of any 1-2-3 skip list is  $\theta(\log n)$ . (6 marks)



Q2. Run Dijkstra's algorithm on the network given in the figure starting from vertex  $s = A$ . Show the steps in running the algorithm, you do not need to draw the graph repeatedly, just write which is the next vertex picked and which labels get updated from what value to what value at each step. (10 marks)

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Q6. Given a sequence of  $n$  integers, a pair of elements  $x$  and  $y$  are called an *inversion* in this sequence if  $x > y$  but  $x$  occurs before  $y$  in the sequence. Give an  $O(n \log n)$  algorithm for finding the number of inversions in a sequence. [Hint: Modify merge sort]. (12 marks)