COL 202 Nov 23

Major Exam

1. You are NOT allowed to stand up or leave the seat at the end of exam till all answer scripts are collected and counted.
2. You are NOT allowed to leave the exam hall during the exam period unless on medical emergency.
3. Calculators and phones are NOT allowed.
4. You are NOT allowed to ask any questions during the exam. If in doubt, make (and state) your assumptions.

Q1. (5 marks) Solve the recurrence

$$(n+1)(n+2)a_{n+2} - 3(n+1)a_{n+1} + 2a_n = 0,$$

with the initial conditions $a_0 = 2, a_1 = 3$.

Q2.(5 marks) Consider the situation where n brother/sister pairs sit in a row of $2n$ chairs for $n \geq 2$. In how many ways can the children sit down so that no brother/sister pair is sitting next to each other?

Q3.(5 marks) Find a number $y \in \{0, 1, \ldots, 112\}$ such that $11^{112111} \equiv y (\text{mod } 113)$. Note that 113 is prime. Show how you obtain y? Any brute force calculation will not fetch marks.

Q4.(5 marks) The circumference of a circle is divided into 36 sectors to which the numbers $1, 2, 3, \ldots, 36$ are assigned in some arbitrary manner. Show that there are three consecutive sectors such that the sum of their assigned numbers is at least 56.

Q5.(5 marks) Let a_r denote the number of ways to divide r identical marbles into four distinct piles so that each pile has an odd number of marbles that is larger than or equal to three. Give a closed-form expression for a_r.

Q6.(3 marks) Let G be a simple planar graph which is also bi-partite. Prove that the number of edges in G is at most $2n - 4$ edges, where n is the number of vertices in the graph.

Q7. (3 marks) Suppose we are working with numbers in the field F_{17}, i.e., $\{0, 1, \ldots, 16\}$. How many distinct polynomials $p(x)$ of degree at most 5 are there such that $p(0) = 8$ and $p(1) = 7$? Give reasons.

Q8. (4 marks) Suppose G is a simple graph with 100 vertices and 65 edges. What is the maximum number of connected components in G? Give reasons.

$(n+2)t^2 - 3(n+1)t + 2 = 0$