COL351: Analysis and Design of Algorithms (CSE, IITD, Semester-I-2016-17) Major

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- You may use any of the following known NP-complete problems to show that a given problem is NP-complete: 3-SAT, INDEPENDENT-SET, VERTEX-COVER, SET-COVER, HAMILTONIAN-CYCLE, HAMILTONIAN-PATH, SUBSET-SUM, 3-COLORING.

There are 6 questions for a total of 40 points.

1. Solve the following questions:

   (a) (1 point) State true or false (reason not required): In any undirected graph, there is always an even number of vertices that have odd degree.

   (b) (1 point) State true or false (reason not required): The number of strongly connected components in any directed graph G with n vertices and n edges is $< n$.

   (c) (2 points) Solve the following recurrence relation and give the exact value of $T(n)$.

$$T(n) = \begin{cases} T(n-1) & \text{if } n > 1 \text{ and } n \text{ is odd} \\ 2 \cdot T(n/2) & \text{if } n > 1 \text{ and } n \text{ is even} \\ 1 & \text{if } n = 1 \end{cases}$$

   (d) (1 point) State true or false (reason not required): Let G be a weighted graph with distinct edge weights and let G' be a graph obtained from G by increasing the weight of every edge of G by 10. The minimum spanning tree of G is the same as the minimum spanning tree of G'.

   (e) (2 points) Recall the Job Scheduling problem that was discussed in class. In this problem n jobs are given each with a duration and a deadline and the goal is to minimize the maximum lateness. Consider a variant of this problem where the goal is to minimize the *sum* of lateness of all jobs. That is, find a schedule such that the *sum* of lateness of jobs is minimized. Show that the greedy algorithm of doing jobs in non-decreasing order of deadlines does not work for this variant of the job scheduling problem.

   (f) (1 point) State true or false (reason not required): The Dijkstra's algorithm when executed on the graph below with starting vertex s returns the correct shortest path from s to all other vertices.