Computer Vision (COL780) Minor I August 27, 2016

Please do Q1, Q2 and Q3 in the stipulated 1 hr. Please submit the solutions of the remaining on Moodle by midnight today.

---

1. Please derive the optic flow equation

$$uI_x + vI_y + I_t = 0$$

clearly stating all assumptions.

2. Suppose we take a first order approximation of the optic flow as

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} + \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + O(x^2, xy, y^2)$$

Please indicate (with reasons) under what conditions may the approximation be valid.

3. Derive a linear least-squares solution to estimate the above approximation. Please indicate all assumptions that are implicit in the process.

4. If $U = (U_1, U_2, U_3)$ and $\Omega = (\Omega_1, \Omega_2, \Omega_3)$ are the translational and rotational velocities of a 3D object point, and $P = (X, Y, Z)$ is the corresponding position vector, then the object velocity can be written as $V = -U - \Omega \times P$. Show that under the pin-hole camera model $x = fX/Z$ and $y = fY/Z$, where f is the pin-hole focal length and (x, y) is the image projection of (X, Y, Z), the optic flow components can be expressed as

$$\begin{aligned} u &= -U_1 f/Z - \Omega_2 f + \Omega_3 y - x(-U_3/Z - \Omega_1 y + \Omega_2 x) \\ v &= -U_2 f/Z - \Omega_3 x + \Omega_1 f - y(-U_3/Z - \Omega_1 y + \Omega_2 x) \end{aligned}$$

5. Conclude that

$$\begin{aligned} u_x &= U_3/Z + U_1 Z_x f/Z^2 &= V_z + V_x Z_X \\ u_y &= \Omega_3 + U_1 Z_y f/Z^2 &= \Omega_3 + V_x Z_Y \\ v_x &= -\Omega_3 + U_2 Z_x f/Z^2 &= -\Omega_3 + V_y Z_X \\ v_y &= U_3/Z + U_2 Z_y f/Z^2 &= V_z + V_y Z_Y \end{aligned}$$

where $V_z = U_3/Z$, $V_x = U_1/Z$, $V_y = U_2/Z$, $Z_X = Z_x f/Z$ and $Z_Y = Z_y f/Z$.

6. It is well known that the velocity tensor gradient can be decomposed uniquely as

$$\begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} = \frac{\mathrm{div}\mathbf{v}}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{\mathrm{curl}\mathbf{v}}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + \frac{\mathrm{def}\mathbf{v}}{2}\begin{bmatrix} \cos 2\mu & \sin 2\mu \\ \sin 2\mu & -\cos 2\mu \end{bmatrix}$$

where

$$\begin{aligned} \mathrm{div}\mathbf{v} &= (u_x + v_y) \\ \mathrm{curl}\mathbf{v} &= -(u_y - v_x) \\ (\mathrm{def}\mathbf{v})\cos 2\mu &= (u_x - v_y) \\ (\mathrm{def}\mathbf{v})\sin 2\mu &= (u_y + v_x) \end{aligned}$$

$\tan 2\mu =$

are scalar differential invariants and are independent of the choice of the coordinate system. Verify the above relationships.

7. Let $\mathbf{Q}$ be camera look-at direction and the vector quantities

$$\mathbf{A} = \frac{U - U \cdot \mathbf{Q}}{Z} \text{ and } \mathbf{F} = \frac{f\nabla Z}{Z}$$

represent the translational velocity of the object parallel to the image plane and the depth gradient respectively, both scaled by the depth Z. The magnitude of the depth gradient, $|\mathbf{F}|$, represents the slant of the object surface (angle between the surface normal and the view direction): and the tilt angle $\angle\mathbf{F}$ specifies the direction in the image of increasing distance. Show that,

$$\begin{aligned}
\text{div}\mathbf{v} &= 2U \cdot \mathbf{Q}/Z + \mathbf{F} \cdot \mathbf{A} \\
\text{curl}\mathbf{v} &= -2\Omega \cdot \mathbf{Q} + |\mathbf{F} \times \mathbf{A}| \\
\text{def}\mathbf{v} &= |\mathbf{F}||\mathbf{A}| \\
\mu &= (\angle\mathbf{A} + \angle\mathbf{F})/2
\end{aligned}$$

8. Comment on the following:

   (a) Argue that the deformation component encodes the orientation of the surface whereas the divergence component can provide an estimate of time to collision.

   (b) If there is no component of motion parallel to the image plane, can you determine the time to collision? Can you determine bounds on time to collision for general motion?

   (c) $\mathbf{A}$ and $\mathbf{F}$ come together in the above equations signifying an inherent *bas-relief* ambiguity. A nearby shallow object will produce the same effect as a far away deep structure.

   (d) In general depth gradient can be recovered only up to an unknown scale. However, can some knowledge of ego-motion (self motion) help determine the surface orientation (consider, for example the orientation of the runway when landing a plane)?