$1+\omega^2$

Question 5. *(4 marks)* A model of a TCP loop under overload conditions (please see Fig. 1a.) is given by the open-loop transfer function,

$$G(s) = \frac{k}{s}e^{-\tau s},$$

where the queuing dynamics are modeled by an integrator, the TCP window control is a time delay τ, and the controller is simply a proportional controller ($k > 0$). Using the Nyquist stability criterion, determine the relation between k and τ for the closed-loop system to be stable.

Question 6. *(3 marks)* The equation of motion of a mechanical system is $\ddot{x} + \dot{x} + x = f$. If the external force $f(t) = 4\sin 3t$, what is the maximum value of $x(t)$ at steady-state?

Question 7. *(2 marks)* The transfer function from steering angle to lateral velocity for a simple vehicle model is,

$$G(s) = \frac{av_0 s + v_0^2}{bs},$$

where v_0 is the velocity of the vehicle and $a, b > 0$. The transfer function has a zero at $s = -v_0/a$. In normal driving ($v_0 > 0$) this zero is in the left-half plane, but is in the right-half plane when driving is in reverse ($v_0 < 0$). For both these cases, calculate and sketch the lateral velocity response to a step change in the steering angle.

Question 8. *(5 marks)* The goal of vertical takeoff and landing (VTOL) aircraft is to achieve operation from relatively small airports and yet operate as a normal aircraft in level flight. An aircraft taking off in a form similar to a missile (on end) is inherently unstable (similar also to an inverted pendulum). A control system using adjustable jets can control the vehicle. The aircraft dynamics are given by (please see Fig. 1b.),

$$G(s) = \frac{1}{s(s-1)},$$

and the controller transfer function is,

$$G_c(s) = K\frac{s+2}{s+10}.$$

Determine the range of gain for which the system is stable. In particular, determine the gain K for which the system is marginally stable and the roots of the characteristic equation for this value of K.

Question 9. *(6 marks)* A computer uses a printer as a fast output device. We desire to maintain accurate position control while moving rapidly through the printer. Consider a system with unity feedback and a transfer function for the motor and amplifier of

$$G(s) = \frac{0.15}{s(s+1)(5s+1)}.$$

Design a lead compensator aimed at achieving a gain crossover frequency of 0.5 rad/s and a phase margin of 30°.

$\sin \omega t$

$\frac{e^{j\omega t} - e^{-j\omega t}}{2}$

$x(0)$

EEL 301 MAJOR TEST

II Semester, 2013-14

Duration: 2 hours

Total Marks: 40

Instructions

1. Show all relevant steps clearly and briefly.
2. If needed, make suitable assumptions. But, state them clearly.
3. Please note reference to following figures in some of the questions,

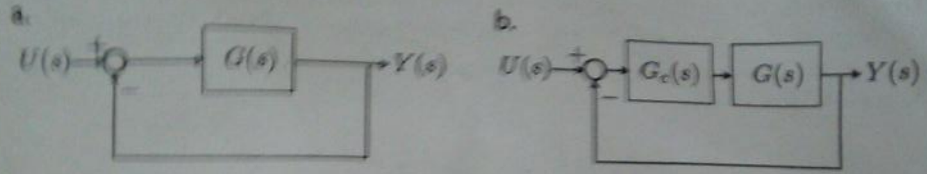

Figure 1: Please note reference to following figures in some of the questions.

Question 1. *(6 marks)* Consider a unity feedback system (Fig. 1a.) with the open loop transfer function $G(s) = 4/(s(s+2))$. Design a lead compensator $G_c(s)$ aimed at achieving $K_v = 20$ and phase margin $\geq 50^\circ$. Sketch the Bode plots of the uncompensated and compensated systems, i.e., of $G(s)$ and $G(s)G_c(s)$.

Question 2. *(4 marks)* Consider a lag-lead compensator with the transfer function,

$$G_c(s) = K_c \frac{(s+1/T_1)(s+1/T_2)}{(s+\beta/T_1)(s+1/(\beta T_2))}.$$

Show that at frequency $\omega = 1/\sqrt{T_1 T_2}$, the phase angle of the compensator becomes 0°. (Hint: $tan^{-1}x + tan^{-1}y = tan^{-1}((x+y)/(1-xy))$.)

Question 3. *(5 marks)* Using the Nyquist stability criterion, investigate the stability of a closed-loop system (Fig. 1a.) with the following open-loop transfer function,

$$G(s) = K\frac{s+3}{s(s-1)}, K > 0.$$

Question 4. *(5 marks)* Consider the unity feedback system shown in Fig. 1a. with

$$G(s) = K\frac{s+a}{s^2(s+3.6)},$$

where a can be 10, 1, or 0.1. For each of these three values of a, plot the root-locus of the closed-loop system as K is varied from 0 to ∞.