EEL 316 Minor Test I Semester II 2013-2014

Answer all questions (Q.1: 20 marks, Q.2: 20 marks) Full Marks: 40

1. A real-valued baseband pulse $g(t)$ with support $[0, T]$ is given by

$$g(t) = A\cos\left(\frac{\pi t}{T} - \pi\alpha\right)\text{rect}\left(\frac{t-(T/2)}{T}\right), \quad A > 0, \quad 0 < \alpha < \frac{1}{2}.$$

Let $h(t)$ be the impulse response of a filter matched to $g(t)$, satisfying the condition $h((1-\alpha)T) = A/2$. Let $y(t) = h(t) \star g(t)$.

(a) Find $h(t)$ and sketch its plot, labeling the relevant portions. [6]

(b) Find the matched filter output $y(t)$ in the range $0 \le t \le T$. Find the value of α for which $y(T/2) = 0$. [6]

(c) For the value of α obtained in (b), find $y(t)$ in the range $T < t \le 2T$. [4]

(d) For the value of α obtained in (b), find $|H(f)|$, where $H(f)$ is the transfer function of the matched filter. [4]

2. Consider the case of binary signaling over an AWGN channel in a bit interval $[0, T]$ with waveforms $s_1(t)$ (for symbol '1') and $s_0(t)$ (for symbol '0'), where the received signal is given by

$$x(t) = \begin{cases} A\left(\frac{t^2}{\alpha^2 T^2} - 1\right)\text{rect}\left(\frac{t-(T/2)}{T}\right) + w(t) & \text{if symbol '1' is transmitted,} \\ B\left(1 - \frac{4}{T^2}\left(t - \frac{T}{2}\right)^2\right)\text{rect}\left(\frac{t-(T/2)}{T}\right) + w(t) & \text{if symbol '0' is transmitted,} \end{cases}$$

$0 \le t \le T$, $A > 0$, $B > 0$, $0 < \alpha < 1$. The additive noise $w(t)$ is a real-valued zero-mean white Gaussian random process with p.s.d. $N_0/2$. The MAP receiver makes the decision

$$\int_0^T x(t)h(t)dt \underset{0}{\overset{1}{\gtrless}} \lambda_{MAP}.$$

The apriori probability of occurrence of symbol '0' is p_0. The waveforms $s_1(t)$ and $s_0(t)$ are *orthogonal*, and $s_1(t)$ and $s_0(t)$ have the same energy. $h(t)$ is chosen so as to minimize P_e and satisfies $h(0) = -A/2$.

(a) Find the value of α. Find B (in terms of A). [6]

(b) Sketch the plots of $s_1(t)$ and $s_0(t)$, labeling the relevant portions in terms of A, T, and t. Find $h(t)$ in terms of A, T, and t. [4]

(c) If $\lambda_{MAP} = \frac{N_0}{2}\ln\left(\frac{2}{3}\right)$, then calculate p_0. [4]

(d) For the values of the parameters obtained in (a), (b), and (c), find P_e in terms of A,T, and N_0. Calculate P_e when $A^2T = 9N_0$. [6]

Some Formulae

- If $X \sim \mathcal{N}(0,1)$, then its p.d.f.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty, \quad \text{and} \quad \Pr[X > x] = \int_x^\infty f_X(y)dy = Q(x) = 1 - Q(-x)$$

- $\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \le \frac{T}{2}, \\ 0 & \text{if } |t| > \frac{T}{2}, \end{cases}$ $\qquad \text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$

- Fourier Transform pairs:

$$\text{rect}\left(\frac{t}{T}\right) \leftrightarrow T\,\text{sinc}\,(fT), \quad \exp(j2\pi f_0 t) \leftrightarrow \delta(f - f_0), \quad G(t) \leftrightarrow g(-f)$$

- MAP receiver:

$$p_1 \exp\left\{-\frac{1}{2}\left(\frac{y - m_1}{\sigma}\right)^2\right\} \underset{0}{\overset{1}{\gtrless}} p_0 \exp\left\{-\frac{1}{2}\left(\frac{y - m_0}{\sigma}\right)^2\right\}$$

$$\lambda_{MAP} = \frac{(m_1 + m_0)}{2} - \frac{\sigma^2}{(m_1 - m_0)} \ln \frac{p_1}{p_0}$$

- $Q(x) \approx \dfrac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x \ge 2.5$