EEL 781 Major Marks: 100 Time: 120 Minutes

Q. 1. Use a Hopfield net to build an A/D converter. Given an analog input x, the task is to find N bits $V_0, V_1, \ldots, V_{N-1}$, so that the binary representation formed with the N outputs is as close as possible to the input value. V_0 is the LSB and V_{N-1} is the MSB. Assume that the input x is a real number in the range 0 to 2^N.

(a) Suggest a Lyapunov or energy function for the task. *Hint:* What is the representation error ?

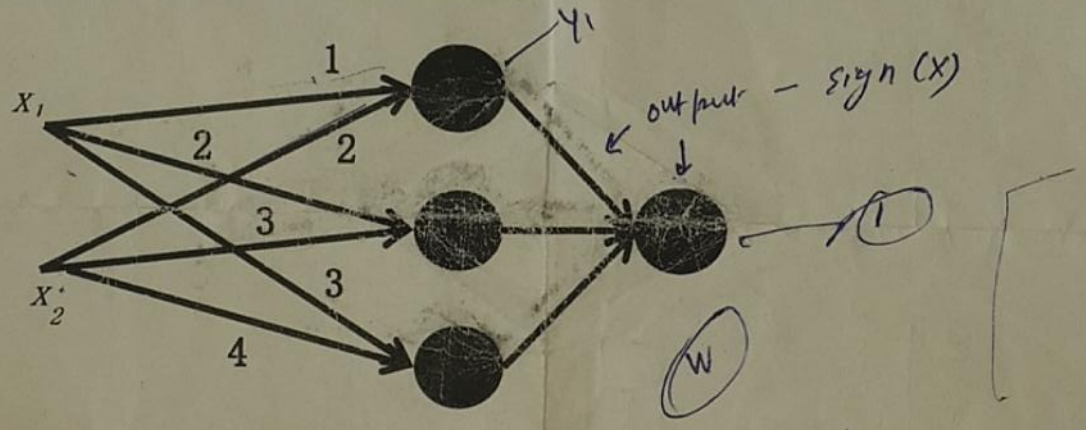
(b) Determine the weights and external current (offset) input J_i for each neuron.

(20)

Q. 2. A Neural Network is shown in Fig. 2 that implements a classifier. Each neuron in the network uses a sigmoid activation function of the form

$$y = 1/1 + exp(-net)$$

The output neuron has a sgn activation function, i.e. y = sign(x) (with values -1 and 1). $1 + exp(-net)$. In order to keep the comparison fair, we would like to use a kernel function for the SVM that has a one-to-one correspondence with the neural network's first layer. Note that the first layer has mapped the inputs into a larger number of neuron outputs. Determine the map $\phi(x)$ for the neural net. If we have two input patterns (0.5, -1) and (1, 1), determine the kernel matrix of the SVM.



(20)

Q. 3. A set of M vectors x^i, i = 1, 2, ..., M is mapped to a higher dimension using a map $\phi(x)$. Determine an expression for (a) the norm of the mean of the image vectors, i.e. $|\mu|$ in terms of the kernel function K, where $K_{ij} = \phi(x^i)^T \phi(x^j)$, where $\mu = \frac{1}{M}\sum_{j=1}^{M} \phi(x^j)$.

(b) Similarly, determine the value of $|\sigma^2|$, i.e. the norm of the variance vector.

(20)

Q. 4. A set of points is provided in the file majq4.txt. Given a new vector (16, 16), determine the projection of this vector onto the kernel principal components. That is, determine the co-ordinates of the vector (16, 16) in the kernel PCA space.

(20)

Q. 5. What would be the steady state weight vector if Oja's rule were used for the patterns (0, 2), (0, -2), (2, 2), (4, 4), (-2, -2), (-4, -4) ?

(10)

Q. 6. Determine the optimal (SVM) linear separating plane in the form

$w^T x + b = 0$

for the points

Class 1: (0, -2), (-2, -2); Class -1: (2, -2), (2, 2), (-2, 2)

(10)