ELL205: Minor 2
Department of Electrical Engineering, IIT Delhi

Time: 24 Hr **Maximum marks:** 30

- **Write your name and entry number on the uploaded answer script, failure to do which will fetch zero marks in the exam.**
- **Brevity in the answers will be given more credit.**
- **Make assumptions if required but state them clearly.**
- **Read the questions carefully before answering them. Answer all the parts of a question in one place. Untidy work will fetch a penalty of -2 marks.**

Undertaking: By attempting this paper you acknowledge that you will abide by the institute Honor Code and the code of conduct for this examination and can be held accountable as per rules established in case of any violation.

1. A continuous time linear system, which is causal and stable, has the following frequency response

$$H(\omega) = \frac{4 + j\omega}{6 - \omega^2 + 5j\omega}$$

   (a) Taking $x(t)$ as input and $y(t)$ as output, write the differential equation of the system. [3]

   (b) Determine the impulse response of the system. [2]

   (c) Find $y(t)$, the output of the system when $x(t) = 2e^{-4t}u(t)$ [3]

2. (a) The following facts are given about a signal $x[n] \leftrightarrow X(\Omega)$ [4]
      1. $x[n] = 0$ for $n > 0$
      2. $x[0] > 0$
      3. $\text{Im}\{X(\Omega)\} = sin\Omega - sin2\Omega$
      4. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega = 3$

      Determine $x[n]$.

   (b) Given the following difference equation governing a causal discrete time LTI system [3]

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

   Determine it's impulse response.

3. Consider a periodic signal $x(t)$, as given in figure below

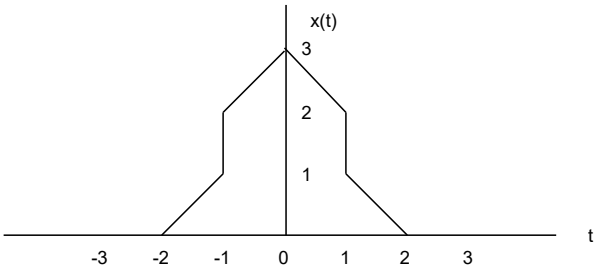

Figure 1: $x(t)$ for Q(3)

One period of $x(t)$ is given as

$$x(t) = \begin{cases} 0, & -3 \le t \le -2 \\ \text{as given in Fig. 1}, & -2 \le t \le 2 \\ 0, & 2 \le t \le 3 \end{cases}$$

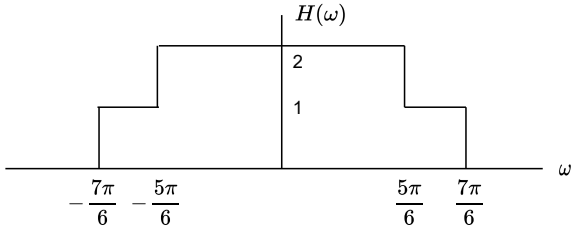

Figure 2: LTI system

(a) Find Fourier series representation for $x(t)$. [4]

(b) If this signal $x(t)$ is given as input to an LTI system having frequency response $H(\omega)$ given in Fig. 2. Determine the output $y(t)$ of the system. [3]

4. Let $x(t) \leftrightarrow X(\omega)$ be Fourier transform pair. Without explicitly evaluating $X(\omega)$, for $x(t)$ as given in figure below

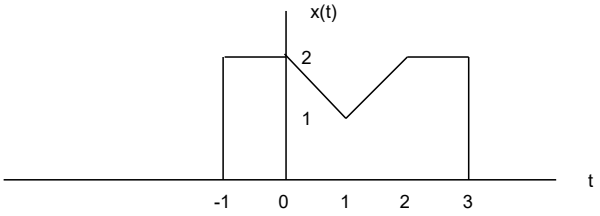

Figure 3: $x(t)$ for Q(4)

(a) Find $\int_{-\infty}^{\infty} X(\omega)d\omega$ [2]

(b) Evaluate $\int_{-\infty}^{\infty} X(\omega)(\frac{2sin\omega}{\omega})e^{j2\omega}d\omega$ [3]

(c) Find and sketch the inverse Fourier transform of $\mathrm{Re}\{X(\omega)\}$ [3]