

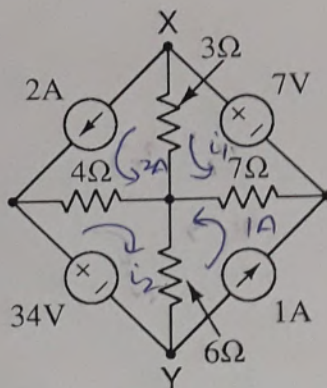
Indian Institute of Technology, Delhi
 ELL 100: Introduction to Electrical Engineering
 Minor 1: February 6, 2018

Instructions:

Answer all questions. Answers written only in the space provided will be checked.

1. In the circuit below find the currents in the four resistors flowing towards the center of the circuit. You may use any method of your choice. What are the voltages at nodes X and Y with respect to the center of the circuit?
 (8 + 4 = 12 marks)

12



$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 34 \end{bmatrix}$$

$$10i_1 - 7 - 7 - 6 = 0$$

$$i_1 = 2A$$

$$10i_2 - 34 + 6 + 8 = 0$$

$$i_2 = 2A$$

current in $3\Omega = 0A$ ✓

$4\Omega = 4A \rightarrow$ ✓

$6\Omega = 3A \downarrow$ ✓ Ans

$7\Omega = 1A \rightarrow$ ✓

$V_x = 0V$ ✓

$V_y = -18V$ ✓

Ans

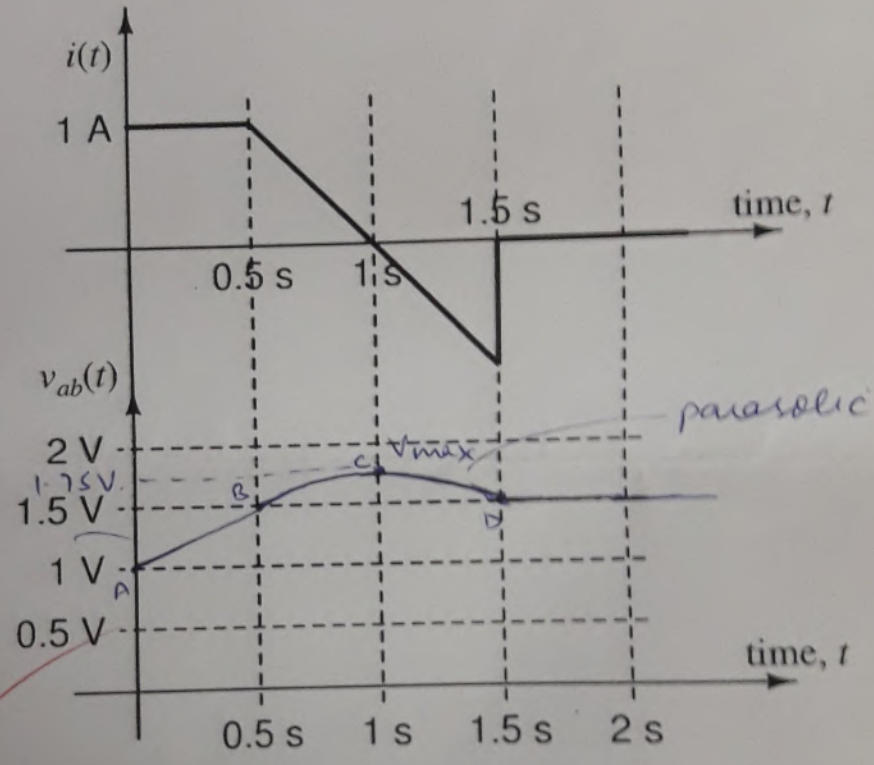
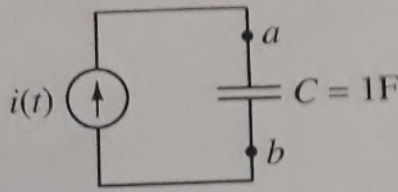
2. A current source $i(t)$ whose profile is shown below is used to charge a capacitor. The initial voltage across the capacitor, $v_{ab}(0)$, is 1 volt.

(a) Sketch the variation of $v_{ab}(t)$. Mark all the important points in the sketch. Sketch your graph only in the grid that has already been made for you.

(b) At what point of time is $v_{ab}(t)$ maximum? What is this maximum value?

(8 + 4 = 12 marks) at $t = 1s$ $V_{as} = 1.75V$

12
12



For maximum value of output V , $i = 0$.
 \Rightarrow at $t = 1$

$$i = C \frac{dv}{dt}$$

$$\int dv = \int i dt$$

$$v - 1 = \frac{1}{2} t^2 - \frac{1}{2} t$$

$$v = \frac{1}{2} t^2 - \frac{1}{2} t + 1$$

~~$i = -2t + 2$~~
 straight line

$$\int_{3/2}^t dv = \int_{3/2}^t (-2t + 2) dt$$

$$v - \frac{3}{2} = -t^2 + 2t - \frac{3}{4}$$

$$v = -t^2 + 2t + \frac{3}{4} \quad (\text{parabolic})$$

$$v(1) = -1 + 2 + \frac{3}{4} = 1.75V$$

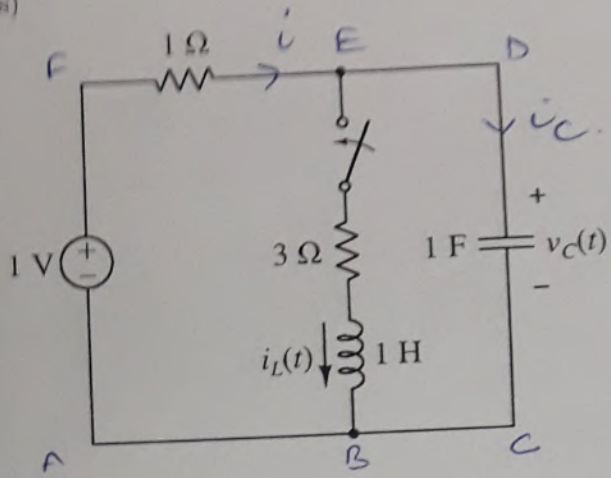
$$v(1.5) = 1.5V$$

3. In the circuit below, the switch is closed at $t = 0$.

- (a) Work out the values of $v_C(0^+)$, $v_C(\infty)$, $i_L(0^+)$ and $i_L(\infty)$.
- (b) Work out the values of $\frac{di_L}{dt}(0^+)$ and $\frac{dv_C}{dt}(0^+)$.
- (c) Use the mesh/loop current method to analyze the circuit. Reduce the two loop equations to one equation in $i_L(t)$.
- (d) Arrive at an expression for $i_L(t)$ using the solution to the equation in part (c), and the initial and final values of i_L and the initial value of $\frac{di_L}{dt}$.

(4 + 2 + 5 + 5 = 16 marks)

16



a)

$$v_C(0^+) = 1V$$

$$v_C(\infty) = \frac{3}{4}V$$

$$i_L(0^+) = 0$$

$$i_L(\infty) = \frac{1}{4}A$$

An

b) At $t=0^+$, the current in loop ABFE is 0
 Voltage across inductor
 $-1 + L \frac{di_L}{dt} = 0$

$$\frac{di_L}{dt} = 1A/s$$

An

at $t=0^+$

$$i_C = C \frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} = 0$$

An

$$c) -1 + i + v_L = 0 \quad i_L = \frac{e \, dv_L}{dt} = \frac{di_L}{dt}$$

$$-1 + i + 3i_L + \frac{di_L}{dt} = 0 \quad \text{--- (1)}$$

$$\nabla \frac{di}{dt} + i = 0 \quad i = i_C + i_L$$

$$\frac{di}{dt} + i - i_L = 0 \quad \text{--- (2)}$$

$$\textcircled{1} \text{ becomes } \frac{di}{dt} + 3\frac{di_L}{dt} + \frac{d^2 i_L}{dt^2} = 0$$

$$i_L = i + 3\frac{di_L}{dt} + \frac{d^2 i_L}{dt^2} = 0$$

$$i_L - 1 + 3i_L + 4\frac{di_L}{dt} + \frac{d^2 i_L}{dt^2} = 0$$

$$\frac{d^2 i_L}{dt^2} + 4\frac{di_L}{dt} + 4i_L = 1 \quad \textcircled{5} \text{ Ans}$$

$$d) \quad i_L = A_1 e^{-2t} + A_2 t e^{-2t} + \frac{1}{4}$$

$$i_L(0^+) = A_1 + \frac{1}{4} = 0 \Rightarrow A_1 = -1/4$$

$$\frac{di_L}{dt}(0^+) = -2A_1 + A_2 = 1$$

$$A_2 = 1/2$$

$$\text{so we get } i_L = -\frac{e^{-2t}}{4} + \frac{t e^{-2t}}{2} + \frac{1}{4} \quad \text{Ans}$$

5

end