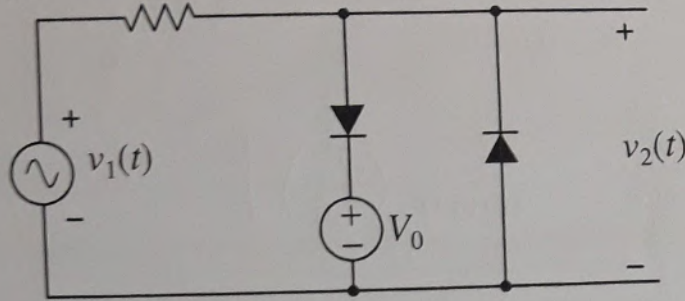


Indian Institute of Technology, Delhi
 ELL 100: Introduction to Electrical Engineering
 Minor 2: March 26, 2018

Instructions:

Answer all questions. Answers written only in the space provided will be checked.

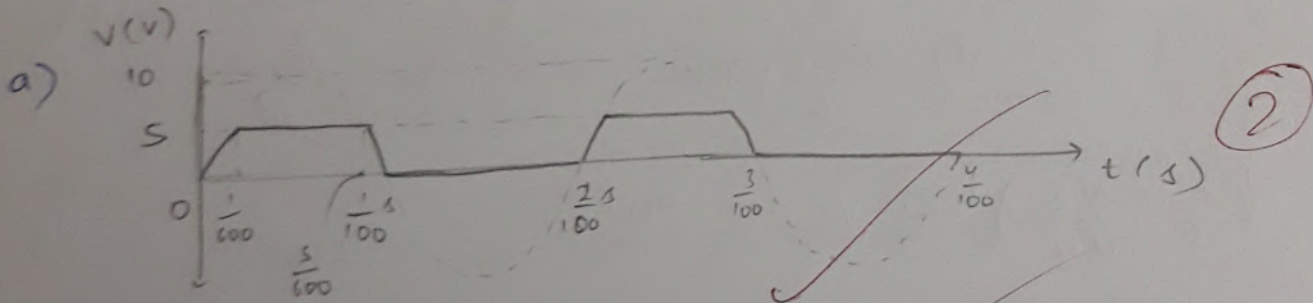
1. In the circuit shown below, the two diodes are ideal with zero cut-in voltage. The voltage source $v_1(t)$ is $10 \sin(100\pi t)$ volts. V_0 is 5 volts.



$100\pi t = \frac{\pi}{6}$

6

- (a) Sketch $v_2(t)$ for two periods, labelling the relevant portions. (2 marks)
- (b) In the first period, find the values of t at which $v_1(t) = V_0$. (2 marks)
- (c) What is the average value of $v_2(t)$? (3 marks)
- (d) What is the RMS (root mean-squared) value of $v_2(t)$? (3 marks)



b) $t = \frac{1}{600} \text{ s}$ and $t = \frac{5}{600} \text{ s}$.

c)

$$\int_0^{\frac{1}{600}} 10 \sin(100\pi t) dt = \frac{10}{100\pi} (\cos(100\pi t)) \Big|_0^{\frac{1}{600}}$$

$$= \frac{1}{10\pi} \left(\frac{\sqrt{3}-1}{2} \right)$$

Total Average = $\frac{\frac{2}{10\pi} \left(1 - \frac{\sqrt{3}}{2} \right) + \frac{4}{600} \times 5}{\frac{1}{100}} = \left(\frac{20}{\pi} \left(1 - \frac{\sqrt{3}}{2} \right) + \frac{20}{6} \right) \text{ V}$

$= 4.186 \text{ V}$ Ans

$$d) \int_0^{\frac{1}{600}} 100 \sin^2(100\pi t) dt$$

$$= 50 \int_0^{\frac{1}{600}} (1 - \cos(200\pi t)) dt$$

$$= 50 \left[t - \frac{1}{200\pi} \sin(200\pi t) \right]_0^{\frac{1}{600}}$$

$$= 50 \left[\frac{1}{600} - \frac{1}{200\pi} \left(\frac{\sqrt{3}}{2} \right) \right]$$

1

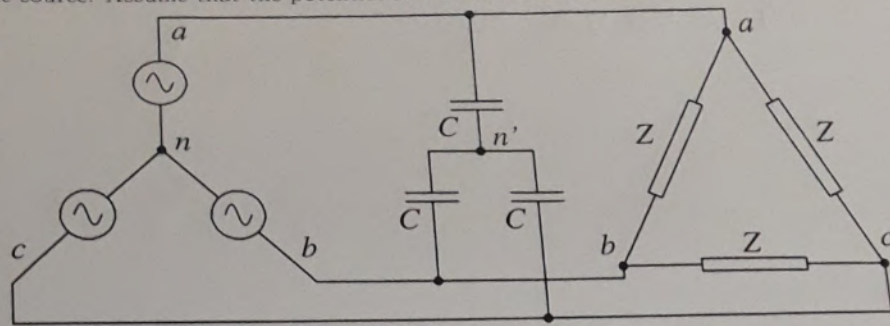
$$RMS = \sqrt{\frac{100 \left[\frac{1}{600} - \frac{1}{200\pi} \left(\frac{\sqrt{3}}{2} \right) \right]^2}{\frac{1}{100}}}$$

$$= \sqrt{\left(\frac{1}{6} - \frac{1}{2\pi \times 2} + \frac{1}{6} \right) 100}$$

$$= \sqrt{\left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi} \right) 100} \text{ V} \quad \underline{\text{Ans}}$$

$$= 4.42 \text{ V}$$

2. A 3-phase 50-Hz voltage source with $V_{an} = 230\angle 0^\circ$ volts, $V_{bn} = 230\angle -120^\circ$ volts, and $V_{cn} = 230\angle 120^\circ$ volts feeds an inductive load with $Z = 5\angle 75^\circ$ ohm, as shown in the schematic below. All voltages are RMS quantities. A capacitor bank is installed to improve the power factor to 1.00, as seen by the source. Assume that the potential at n is the same as at n' .



- (a) What is the three-phase apparent power (active plus reactive) absorbed by the inductive load? (4 marks)
- (b) What is the value of C ? (4 marks)
- (c) What is the three-phase apparent power absorbed by the capacitor bank? (2 marks)
- (d) What is the three-phase apparent power supplied by the source? (2 marks)

By 2 load

$$V_{ab} = V_a - V_b = 230\sqrt{3} \angle 30^\circ$$

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{230\sqrt{3} \angle 30^\circ}{5 \angle 75^\circ}$$

$$= 46\sqrt{3} \angle -45^\circ$$

Power = $(31.74 \angle 75^\circ \text{ kVA}) \times 3 = 95.22 \angle 75^\circ \text{ kVA}$ (4) Ans

By C load

$$V_{an} = 230\angle 0^\circ$$

$$Z = \frac{1}{\omega C} \angle -90^\circ$$

$$I_{an} = \frac{230\angle 0^\circ}{\frac{1}{\omega C} \angle -90^\circ}$$

$$= 230\omega C \angle 90^\circ$$

Power = $3 V_{an} I_{an}$

$$= 3 (230\angle 0^\circ) (230\omega C \angle 90^\circ)$$

$$= 158.7\omega C \angle -90^\circ \text{ kW} = 95.22 \sin 75^\circ$$

$$\omega C = 0.58$$

$$C = \frac{0.58}{2\pi f} = 1.84 \text{ mF}$$

Ans

c) $98 \rightarrow -91.97 \text{ kVAR kW} = -95.22 \sin 75^\circ$

②

d) $+24.64 \text{ k~~VAR~~ kW}$

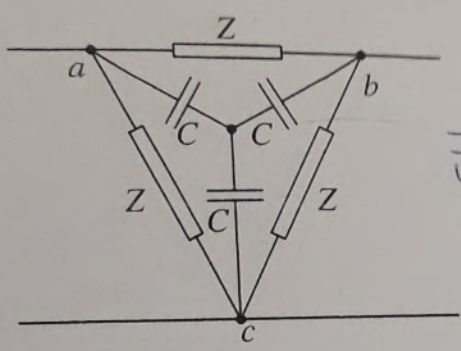
②

$$\frac{5 \angle 75^\circ (\angle -90^\circ)}{0.58 \left(\frac{-3 \angle 90^\circ}{0.58} + 2 \right)}$$

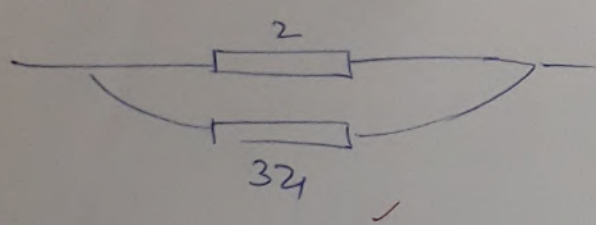
$$\left[3.1 \angle 54^\circ \quad 1.55 \angle 54^\circ \right]$$

(e) The inductive load and the capacitor bank are redrawn below. Visualize the schematic as a two-port network, with the two ports as ac and bc. Calculate the Z-parameters of the two-port network. (4x2 = 8 marks)

8/8

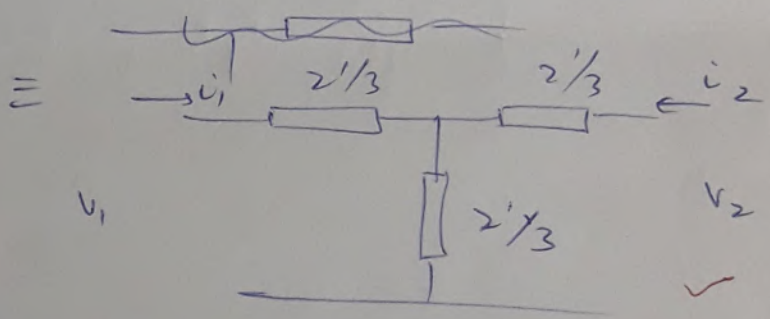
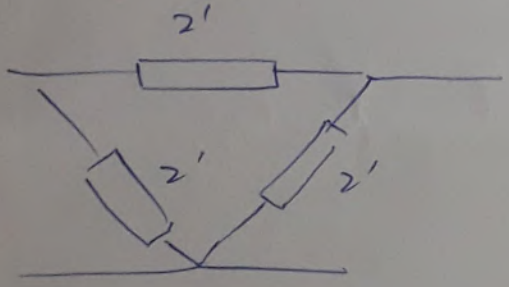


$$\frac{-j}{\omega C} = Z$$



~~$$\frac{2Z \cdot \frac{3Z}{2}}{2Z + \frac{3Z}{2}}$$~~

$$\frac{3Z \cdot 2}{3Z + 2} = Z'$$



$$\begin{bmatrix} V_2 \\ V_1 \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\frac{Z' \cdot 2}{3Z' + 2} = \frac{-jZ}{\omega C} \left(\frac{-3j + 2}{\omega C} \right)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2Z'/3 & Z'/3 \\ Z'/3 & 2Z'/3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

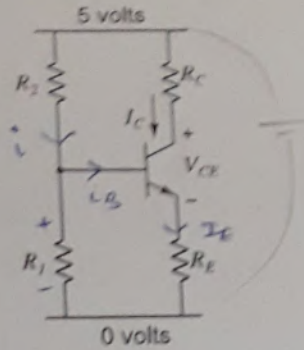
$$Z = \begin{bmatrix} \frac{2Z' \cdot 2}{3Z' + 2} & \frac{Z' \cdot 2}{3Z' + 2} \\ \frac{Z' \cdot 2}{3Z' + 2} & \frac{2Z' \cdot 2}{3Z' + 2} \end{bmatrix} = \begin{bmatrix} \frac{-2jZ}{-3j + 2\omega C} & \frac{-jZ}{-3j + 2\omega C} \\ \frac{-jZ}{-3j + 2\omega C} & \frac{-2jZ}{-3j + 2\omega C} \end{bmatrix}$$

$$\begin{bmatrix} 3.1 \angle 54^\circ & 1.55 \angle 54^\circ \\ 1.55 \angle 54^\circ & 3.1 \angle 54^\circ \end{bmatrix}$$

Ans

3. In the BJT biasing circuit shown below, I_C is to be set to 2 mA, and V_{CE} is to be set to 2 volts. The ratio of R_C and R_E is to be set to 5, i.e., $R_C/R_E = 5$. The β of the device is 200. Assume V_{BE} is 0.7 volts.

4



- What is the value of R_E ? (2 marks)
- What is the value of R_C ? (2 marks)
- The value of $R_1 \parallel R_2$ is 100 k Ω . What are the values of R_1 and R_2 ? (3 + 3 = 6 marks)

$$-5 + i_C R_C + V_{CE} + i_E R_E = 0$$

$$-3 + \cancel{V_{BE}} + i_C R_E + i_E R_E = 0$$

$$R_E = \frac{3}{i_C + i_E} = \frac{3}{12.0 \mu A}$$

a) $R_E = 0.25 \text{ k}\Omega$ Am

b) $R_C = 5 R_E = 1.25 \text{ k}\Omega$ Am

$$0 - i R_1 + 0.7 - i_E R_E = 0$$

$$i R_1 = 0.7 - \frac{1}{4}(2) = 0.2 \text{ V}$$

considering i_B to be small.

$$-5 + i(R_1 + R_2) = 0$$

$$i R_1 + i R_2 = 5$$

$$i R_2 = 4.8$$

$$\frac{R_2}{R_1} = 24$$

if

$$\frac{R_1 R_2}{R_1 + R_2} = 100$$

$$\left(\frac{0.2}{i}\right) \left(\frac{4.8}{i}\right) \left(\frac{i}{5}\right) = 100 \Rightarrow i = 0.192 \text{ mA}$$

$$R_1 = \frac{0.2}{0.192} = 1.042 \text{ k}\Omega \quad \text{Ans}$$

$$R_2 = 25 \text{ k}\Omega \quad \text{Ans}$$

Current in the branch.

~~Answer~~

Since 0.01 mA can be considered small in respect to 0.2 mA , our assumption was valid.