ELL205: Major examination
Department of Electrical Engineering, IIT Delhi

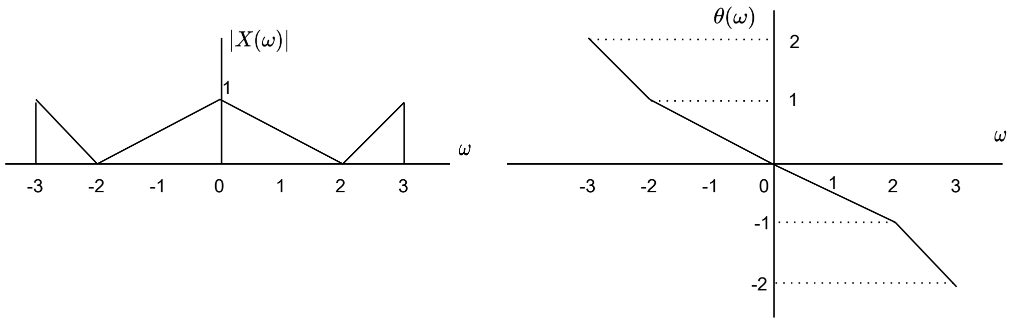
Time: 2 Hr **Maximum marks:** 40

- **Write your name and entry number on the uploaded answer script, failure to do which will fetch zero marks in the exam.**
- **Brevity in the answers will be given more credit.**
- **Make assumptions if required but state them clearly.**
- **Read the questions carefully before answering them. Answer all the parts of a question in one place. Untidy work will fetch a penalty of -2 marks.**

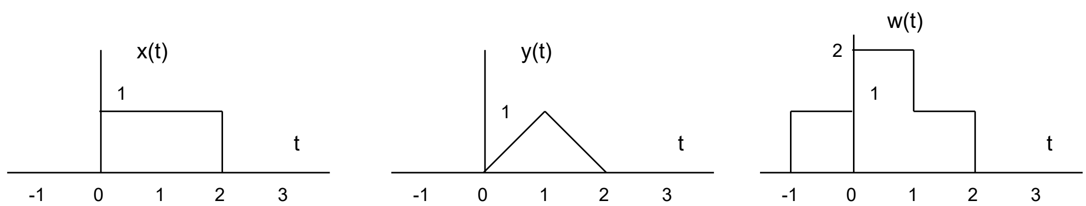
Undertaking: By attempting this paper you acknowledge that you will abide by the institute Honor Code and the code of conduct for this examination and can be held accountable as per rules established in case of any violation.

1. (a) Find Fourier transform of the following signals: [4]
   i. $x(t) = |t|e^{-3|t|}$
   ii. $x(t) = (te^{-4t}sin10t)u(t)$

   (b) Find the inverse Fourier transform for $X(\omega) = |X(\omega)|e^{j\theta(\omega)}$, given in figure below. [5]



2. (a) Consider a LTI system whose response to $x(t)$ is $y(t)$. That is $y(t) = (Rx)(t)$. Determine its response to the input $w(t)$ [3]



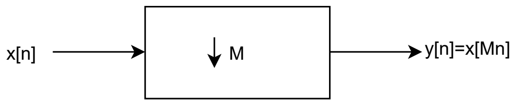
   (b) Give an example of time invariant system and an aperiodic input $x(t)$ such that, corresponding response is periodic. [2]

   (c) Consider a LTI system with input $x(t)$ and output $y(t)$ are related by the following equation [3]

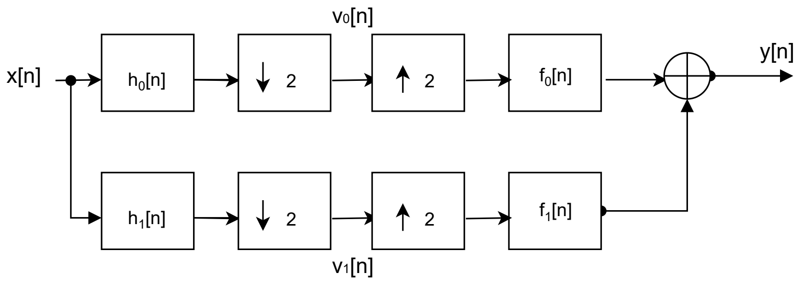
$$y(t) = \int_{-\infty}^{t} e^{-5(t-\tau)}x(\tau - 10)d\tau$$

   Determine its impulse response $h(t)$.

3. (a) Determine the DTFT of $y[n]$ in terms of DTFT of the input $x[n]$ for the following system [3]



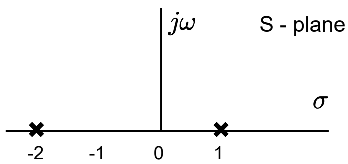
(b) Determine any set of conditions on the filters such that $y[n] = x[n]$ for every input $x[n]$. [5]



Here, $h_0(n)$, $h_1(n)$, $f_0(n)$ and $f_1(n)$ represent impulse responses of LTI systems.

4. (a) Let $x(t)$ be a real signal and $X(s)$ is the algebraic expression of its Laplace transform. Also, $X(s) = \frac{P(s)}{Q(s)}$, where $P(s)$ and $Q(s)$ are finite degree polynomials in s. Comment on the following statement: Poles and zeros of $X(s)$ occur in conjugate pair. Further the polynomials $P(s)$ and $Q(s)$ have all real coefficients. [3]

(b) Let $x(t)$ be a real signal with Laplace transform denoted as $X(s)$. The pole zero plot is given as in figure below [3]



Also we are given $X(0) = 2$ and the fact that $x(t)e^t$ is absolutely integrable. Determine $x(t)$.

5. (a) Consider a discrete time system for which $Y(\Omega)$ the DTFT of the output $y[n]$ is related to the DTFT of the input $x[n] \leftrightarrow X(\Omega)$ in the following way [5]

$$Y(\Omega) = \int_{\Omega-\frac{\pi}{4}}^{\Omega+\frac{\pi}{4}} X(\Omega)d\Omega$$

Find an expression of $y[n]$ in terms of $x[n]$.

(b) Let the output $y[n]$ of a causal discrete time LTI system be related to the input $x[n]$ by the following equation [4]

$$y[n] - \frac{1}{2}y[n-1] = \sum_{k=-\infty}^{\infty} x[k]w[n-k] - x[n]$$

where $w[n] = (\frac{1}{3})^n u[n] + 4\delta[n]$. Determine the frequency response and impulse response of the system.

List of Formula

Convolution $y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$; $y[n]=\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

CTFS $x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$; **DTFS** $x[n]=\sum_{k=\langle N\rangle} a_k e^{jk\frac{2\pi}{N}n}$

$a_k=\frac{1}{T}\int_{\langle T\rangle} x(t)e^{-jk\frac{2\pi}{T}t}$; $a_k=\frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\frac{2\pi}{N}n}$

CTFT $X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$. **DTFT** $X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

$x(t)=\frac{1}{2\pi}\int X(\omega)e^{j\omega t}\,d\omega$; $x[n]=\frac{1}{2\pi}\int X(\Omega)e^{j\Omega n}\,d\Omega$

Properties

(i) $a_1x_1(t)+a_2x_2(t)\leftrightarrow a_1X_1(\omega)+a_2X_2(\omega)$; (i) $a_1x_1[n]+a_2x_2[n]\leftrightarrow a_1X_1(\Omega)+a_2X_2(\Omega)$

(2) $x(t-t_0)\leftrightarrow e^{-j\omega t_0}X(\omega)$; ② $x[n-n_0]\leftrightarrow e^{-j\Omega n_0}X(\Omega)$

$e^{j\omega_0 t}x(t)\leftrightarrow X(\omega-\omega_0)$; $e^{j\Omega_0 n}x[n]\leftrightarrow X(\Omega-\Omega_0)$

(3) $\overline{x(t)}\leftrightarrow \overline{X(-\omega)}$; ③ $\overline{x[n]}\leftrightarrow \overline{X(-\Omega)}$

(4) $x(-t)\leftrightarrow X(-\omega)$; ④ $x[-n]\leftrightarrow X(-\Omega)$

⑤ $x[\frac{n}{2}]\leftrightarrow X(2\Omega)$

(5) $x(at)\leftrightarrow \frac{1}{|a|}X(\omega/a)$;

(6) $X(t)\leftrightarrow 2\pi x(-\omega)$; ⑥ $\sum x_1[n]\overline{x_2[n]} = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X_1(\Omega)\overline{X_2(\Omega)}\,d\Omega$

(7) $\int x_1(t)\overline{x_2(t)}\,dt=\int X_1(\omega)\overline{X_2(\omega)}\,d\omega$;

(8) $x_1(t)*x_2(t)\leftrightarrow X_1(\omega)X_2(\omega)$; ⑦ $x_1[n]*x_2[n]\leftrightarrow X_1(\Omega)X_2(\Omega)$

(9) $x_1(t)x_2(t)\leftrightarrow \frac{1}{2\pi}\int X_1(\sigma)X_2(\omega-\sigma)\,d\sigma$; ⑧ $x_1[n]x_2[n]\leftrightarrow \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X_1(\sigma)X_2(\Omega-\sigma)\,d\sigma$

Laplace Transform $X(s)=\int_{-\infty}^{\infty} x(t)e^{-st}\,dt$

$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$

$e^{-\alpha t}u(t)\leftrightarrow \frac{1}{s+\alpha}$ ROC $Re\{s\}>-Re\{\alpha\}$

$-e^{-\alpha t}u(-t)\leftrightarrow \frac{1}{s+\alpha}$ $Re\{s\}<-Re\{\alpha\}$