ELL205: Minor 1
Department of Electrical Engineering, IIT Delhi

Time: 60 min. **Maximum marks:** 25

- **Write your name and entry number on the uploaded answer script, failure to do which will fetch zero marks in the exam.**
- **Brevity in the answers will be given more credit.**
- **Make assumptions if required but state them clearly.**
- **Read the questions carefully before answering them. Answer all the parts of a question in one place. Untidy work will fetch a penalty of -2 marks.**

Undertaking: By attempting this paper you acknowledge that you will abide by the institute Honor Code and the code of conduct for this examination and can be held accountable as per rules established in case of any violation.

1. Justify whether the given statements are true or false with appropriate reasoning.
   (a) A discrete time LTI system is stable if and only if its step response is absolutely summable. [2]
   (b) A discrete time LTI system is causal if and only if its step response is causal. [3]
   (c) The inverse of a discrete time LTI system if it exists is also an LTI system. [3]

2. Consider the cascade interconnection of three causal LTI systems as shown in fig.1(a). The impulse response $h_2[n]$ is given as $h_2[n] = u[n] - u[n-2]$, and the overall impulse response is as shown in fig. 1(b).

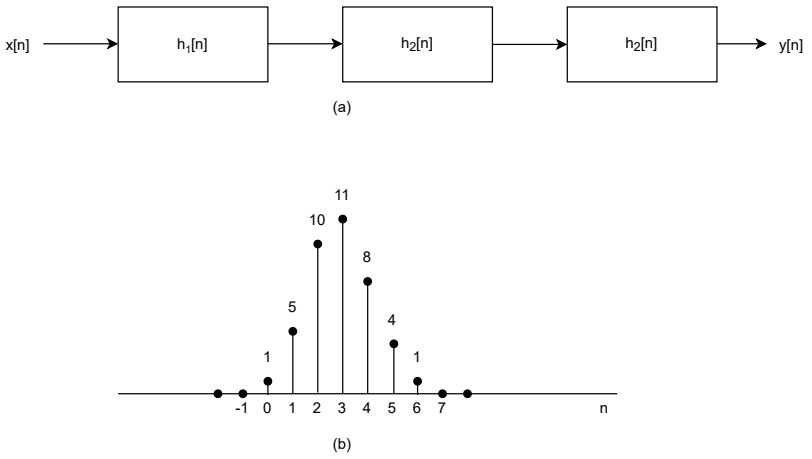

Figure 1: The cascade interconnection

   (a) Find the impulse response $h_1[n]$ [4]
   (b) Find the response of the overall system to the input $x[n] = \delta[n] - \delta[n-1]$ [2]

3. Consider a discrete time causal finite impulse response LTI system $h[n] = \begin{cases} h_n, & n \in \{0, 1, 2, \ldots, M-1\} \\ 0, & \text{otherwise} \end{cases}$.
   Let $x[n] = \begin{cases} x_n, & n \in \{0, 1, 2, \ldots, N-1\} \\ 0, \text{otherwise} \end{cases}$ be an input to the above system for which the output $y[n] = \begin{cases} y_n, & n \in \{0, 1, 2, \ldots, K-1\} \\ 0, \text{otherwise} \end{cases}$, where $h_i, x_i, y_i \in \mathbb{R} \; \forall \; i$, $h_0, x_0, h_{M-1}, x_{N-1} \neq 0$ and $M, N, K \in \mathbb{N}$. Then,
   (a) Find the maximum possible value of K in terms of M and N. [1]

(b) Express the convolution operation to compute the output response $y[n] = x[n] * h[n]$ as a matrix operation given by [2]

$$\mathbf{y} = \mathbf{Hx}$$

where, $\mathbf{y} \in \mathbb{R}^{\mathbf{K\times 1}}$ (column vector) depicts the output $y[n]$ (given above), $\mathbf{x} \in \mathbb{R}^{\mathbf{N\times 1}}$ is the input $x[n]$ (given above) and $\mathbf{H} \in \mathbb{R}^{\mathbf{n_1\times n_2}}$ is the system operator constructed using samples h_n, $n \in \{0, 1, \ldots, M-1\}$. Also find n_1 and n_2 in terms of M and N.

(c) Now, suppose the output response is given as $\mathbf{y} \in \mathbb{R}^{\mathbf{1\times K}}$ (row vector). Express the convolution operation as [2]

$$\mathbf{y} = \mathbf{hX}$$

where, $\mathbf{h} \in \mathbb{R}^{\mathbf{1\times M}}$ denotes the impulse response $h[n]$ and $\mathbf{X} \in \mathbb{R}^{\mathbf{n_1\times n_2}}$ constructed from the input samples $x_n,\ n \in \{0, 1, \ldots, N-1\}$. Also find n_1 and n_2 in terms of M and N.

4. (a) Find the relationship between $x[n]$ and $y[n]$ of the system as shown in fig. 2. [3]

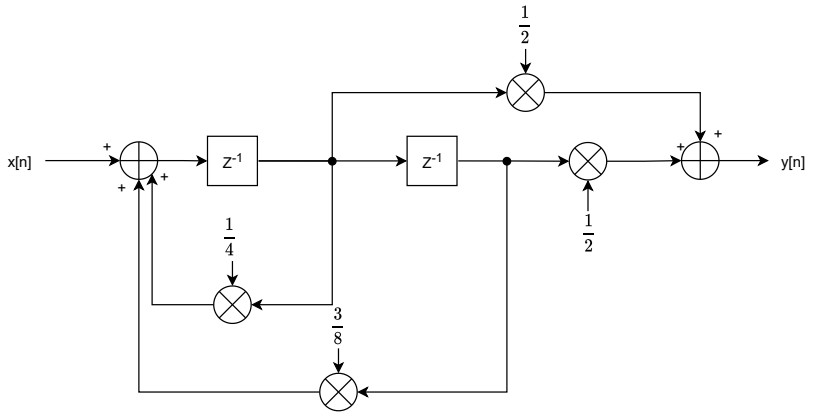

Figure 2: LTI system

(b) Consider a system governed by the following difference equation [3]

$$y[n] + \frac{1}{5}y[n-1] = x[n]$$

and

$$y[-1] = 1$$

Find the closed form expression for the impulse response of the system.