2016EE30511

ELL225, Major Test, Control Engineering I 2017-18/II

Total Time: 2 hour. Maximum marks: 43; Q1:5, Q2:5, Q3:5, Q4: 5, Q5:5, Q6:5, Q7:5, Q8: 4, Q9: 4

- **Write answer all the parts of a question in same place.**

Q1. A second order system is described by following equations:

$$\frac{dx_1}{dt} = x_2(t)$$

$$\frac{dx_2}{dt} = -100\,k_p x_1(t) - 100\,k_d x_2(t) + 100\ r(t)$$

$$y = x_1(t)$$

where y is the output and r is the input. The scalars k_p and k_d are positive.

(a) Find the transfer function G(s) of the system.

(b) Find the region in k_p vs k_d plane (k_p being vertical axis) such that system damping ratio will be $0 < \xi < 1$?

(c) Find the values of k_p and k_d such that $\xi = 0.707$ and undamped natural frequency $\omega_n = 10$ rad/s?

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Q2. The open loop transfer function of a unity negative feedback system is given as

$$G(s) = \frac{k(s+10)^2}{s^3}, \text{ where } k > 0,$$

(a) Draw the Nyquist plot of the system in G(s) plane

(b) Find the range of K for the stability of the system **using Nyquist stability criteria.** Clearly state your arguments.

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Q3. Consider that the open loop transfer function of a unity negative feedback system is

$$G(s) = \frac{e^{-Ts}(as+1)}{s^2}, \quad T \geq 0, a > 0$$

(a) Suppose delay T=0. Determine the value of 'a' so that the phase margin will 45^0? Also find the gain-cross over frequency?

(b) Using results of part (a), find the **maximum** value of delay T for closed-loop stability. (Compute it without using any approximation of e^{-Ts})

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Q4. Consider a SISO (single-input single-output) controllable and observable system (A,B,C) described as

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where x is n-dimensional state vector.

(a) Show that the system (A+BK, B,C) is controllable where K is any non-zero (1xn) row vector?

(b) A state vector z = Mx (where M is a non-singular matrix) is used to define a new state variable model $(\overline{A}, \overline{B}, \overline{C})$. The controllability and observability matrices of (A,B,C) are U and V, respectively. Similarly the controllability and observability matrices of $(\overline{A}, \overline{B}, \overline{C})$ are $\overline{U}$ and $\overline{V}$, respectively. Show that the matrices UV and $\overline{UV}$ shares same set of eigenvalues?

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Q5. Suppose G(s) is the open loop plant transfer function of a unity negative- feedback system where

$$G(s) = \frac{K\left(1 + \frac{1}{Ts}\right)}{s(s+2)(s+3)}, K > 0 \text{ and } T > 0.$$

Using Routh-Hurwitz criterion, find the range of gain K as a function of T such that the system is stable? Comment on the effect of increasing T on the upper limit of gain K for stability.

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Q6. Consider a third order state variable model (A,B,C)

$$\dot{x} = Ax + Bu, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha & -11 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

$$y = Cx, \quad \text{Initial condition } x(0) = x_0$$

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(a) Find the range of scalar α such that the system is controllable?

(b) Suppose, the eigenvalues of matrix A are $-1, -2$ and -3. Determine the initial condition $x(0)$ such that the **zero-input response** will be of the form $x(t) = e^{-t}x(0)$, for $t \geq 0$.

Q7. The Bode plot of a minimum phase transfer function G(s) is shown in the figure below.

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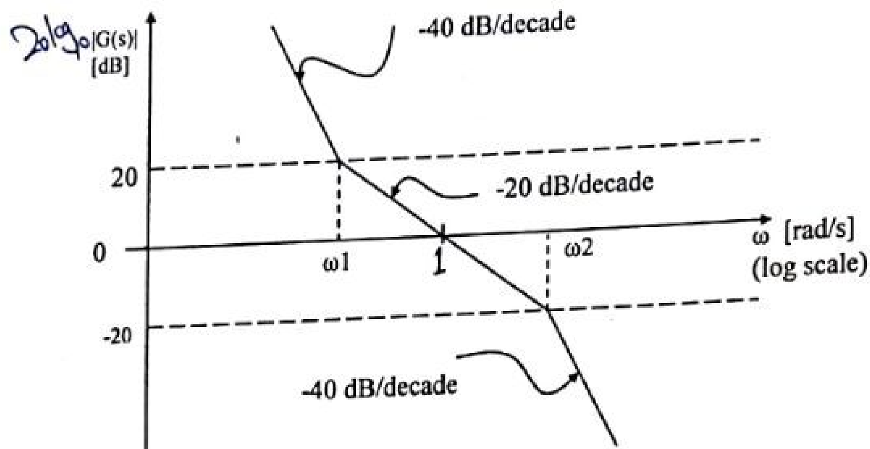

(a) Determine G(s)?
(b) Find a constant K such that the gain cross-over frequency of KG(s) will be 10 rad/sec?
(c) What will be the phase of G(s) in **high** frequency range (>> 100 rad/s)?

Q8. In the figure shown below, D(s) is the compensator and G(s) is given by

$$G(s) = \frac{k}{s^2}$$

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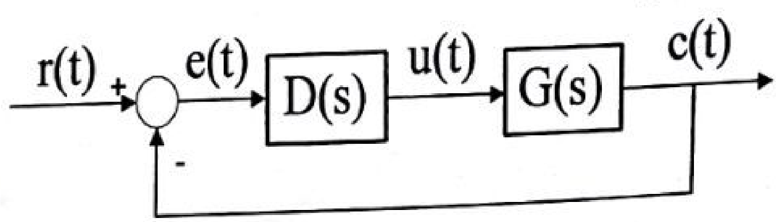

Figure for question 8

(a) Using the results of root locus, design a first order lead compensator **D(s)** and k such that dominant pole will be located at $-1 + j2$. It is assumed that the zero of D(s) is located at $s = -1$.

(b) Determine the static **acceleration** error constant K_a of the system?

$\frac{1}{\omega_1^2} = 20$

$\frac{1}{\omega_2^2} = 1/10$