ELL311: Major Examination
Department of Electrical Engineering, IIT Delhi

Time: 2 Hr | **Maximum marks:** 50

- **Write your name and entry number on the uploaded answer script, failure to do which will fetch zero marks in the exam.**
- **Brevity in the answers will be given more credit.**
- **Make assumptions if required but state them clearly.**
- **Read the questions carefully before answering them. Answer all the parts of a question in one place. Untidy work will fetch a penalty of -2 marks.**
- **If any student is suspected of cheating or copying from others then he/she will be given overall grade 'F' irrespective of whatever marks he/she has obtained in other exams.**

Undertaking: By attempting this paper you acknowledge that you will abide by the institute Honor Code and the code of conduct for this examination and can be held accountable as per rules established in case of any violation.

1. (a) For the circuit given below, find the auto-correlation function and power spectral density of the output $Y(t)$. What is the value of mean power in the output $Y(t)$? Given input $X(t)$ is zero mean with power spectral density [5]

$$S_X(\omega) = \sigma^2 \quad \forall\, \omega$$

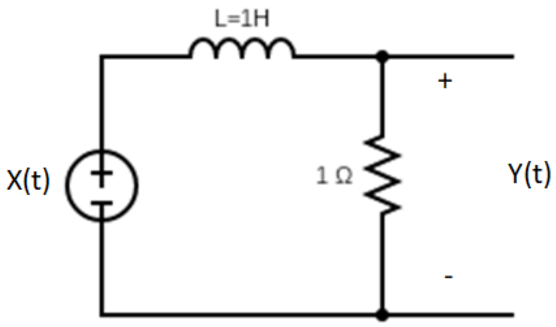

Figure 1: Circuit diagram

   (b) Let $X(t)$ be a zero mean WSS message signal, having power spectral density $S_X(\omega)$ and autocorrelation function $R_X(\tau)$. Find the autocorrelation function and power spectral density of the signal $S(t) = X(t)cos\omega_c t - \hat{X}(t)sin\omega_c t$ where $\hat{X}(t)$ is the Hilbert transform of $X(t)$. [5]

2. (a) Let a sinusoidal signal $m(t) = cos(2\pi * 10^4 t)$ amplitude modulates a carrier, of frequency 1 MHZ, with 50 % depth of modulation. This AM signal is passed through a filter, having frequency response as given in the Fig. 2 Determine the modulation index of the output of the filter. [5]

   (b) Consider a modulated signal $s(t) = (A + m_l(t) + m_r(t))cos\omega_c t + (m_l(t) - m_r(t))sin\omega_c t$ where $m_l(t)$ and $m_r(t)$ are left hand and right hand components of the stereo transmission.Design a system to obtain the components $m_l(t)$ and $m_r(t)$ from $s(t)$. Clearly state all the assumptions and approximations. [5]

3. Making all suitable assumptions, derive the expression for the relationship between the complex envelopes of the band-pass input $x(t)$ and band-pass impulse response $h(t)$ of the LTI system. [10]

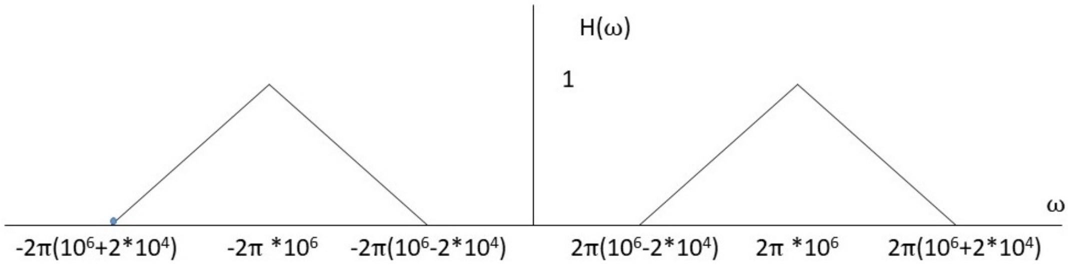

Figure 2: Fig for question 2(a)

4. Consider a nonlinear system, with input $x(t)$ and output $y(t) = a_1x(t) + a_2(x(t))^2 + a_3(x(t))^3$, where a_is are constants. Let an FM signal where message signal $m(t)$ has bandwidth W and peak frequency deviation $\Delta\omega_p$ be passed through this system. The output $y(t)$ is passed through a band-pass filter to produce $s(t)$. Find the lower bound on ω_c (the carrier frequency) in terms of W and $\Delta\omega_p$ so that $s(t)$ still remains an FM signal. What is the bandwidth of the band-pass filter used. [10]

5. Find the minimum sampling frequency ω_s required so that $y(t)$ can be uniquely reconstructed from its samples. Provide mathematical justification. [10]