ELL311: Minor 1
Department of Electrical Engineering, IIT Delhi

Time: 70 minutes **Maximum marks:** 20

- **Write your name and entry number on the uploaded answer script, failure to do which will fetch zero marks in the exam.**
- **Brevity in the answers will be given more credit.**
- **Make assumptions if required but state them clearly.**
- **Read the questions carefully before answering them. Answer all the parts of a question in one place. Untidy work will fetch a penalty of -2 marks.**

Undertaking: By attempting this paper you acknowledge that you will abide by the institute Honor Code and the code of conduct for this examination and can be held accountable as per rules established in case of any violation.

1. For the schematic shown in figure 1, k is a constant which is under the control of the designer and $v(t)$ is a signal which is a function of the message signal $m(t)$, again under the control of the designer and $a, b > 0$ are fixed. Then, find k and $v(t)$ in terms of a, b and $m(t)$, so that the output of the schematic, $y(t)$ is:
   (a) A DSB-SC signal. [2]
   (b) A normal AM signal. [2]

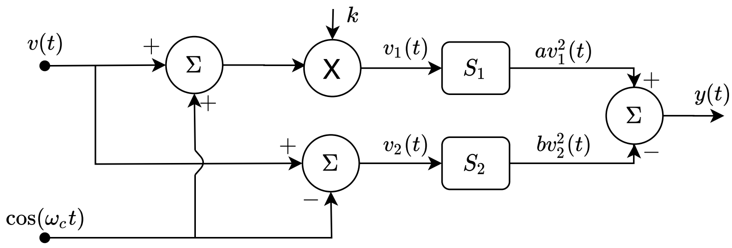

Figure 1

2. Let $x_{1+}(t)$ and $x_{2+}(t)$ be the pre-envelopes of the real band-pass signals $x_1(t)$ and $x_2(t)$ respectively.
   (a) Show that $\int \text{Re}\left[x_{1+}(t)\right] \text{Re}\left[x_{2+}(-t)\right]\, dt = \frac{1}{2}\text{Re}\left[\int x_{1+}(t) x_{2+}(-t)\right]\, dt$ [2½]
   (b) Using the above relation, show that for a band-pass LTI signal with impulse response $h(t)$, the complex envelope, $\widetilde{y}(t)$, of the output $y(t)$ is given by [1½]

$$2\widetilde{y}(t) = \widetilde{x}(t) * \widetilde{h}(t)$$

   where, $\widetilde{x}(t), \widetilde{h}(t)$ are the complex envelopes corresponding to band-pass input signal $x(t)$ and the band-pass impulse response $h(t)$.

3. Let $m(t) = \frac{\omega_0}{2}\left(\frac{\sin(\omega_0 t)}{\pi t}\right)^2$, where $\omega_0 = 4000\pi$ rad/s and let $\omega_c = 40000\pi$ rad/s. Find the closed form expression for LSB-SSB in the time domain. [4]

4. Let $x(t)$ be a complex band-pass signal with the baseband complex envelope given by $\widetilde{x}(t)$. If $\int_{-\infty}^{0} |X(\omega)|^2\, d\omega = \int_{0}^{\infty} |X(\omega)|^2\, d\omega = \pi E < \infty$, where $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$, then derive an expression for the energy of $\widetilde{x}(t)$ in terms of E. [3]

5. Let $m(t)$ be a base-band message signal whose spectra is given as $M(f) = \begin{cases} \cos(2\pi f T_0), & |f| \leq \frac{1}{4T_0} \\ 0, & \text{otherwise} \end{cases}$. Let $x(t)$ be an AM signal, modulated using the message signal $m(t)$ such that,

$$x(t) = A(1 + km(t))\cos(2\pi f_c t + \phi)$$

where, $A > 0, k \in \mathbb{R}, \phi \in (-\pi, \pi]$ and $f_c \gg \frac{1}{4T_0}$. Then,

(a) Find the signal $m(t)$. [3]

(b) Find the complex envelope, $\widetilde{x}(t)$, for the signal $x(t)$. [2]