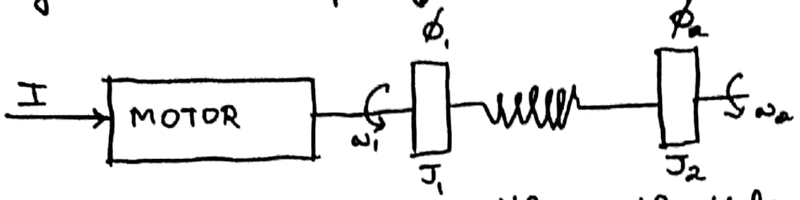
Prof. Shaunak Sen

ELL 333 MINOR TEST 1 MARKS = 15 DURATION = 1 HOUR

1. Consider a system consisting of a motor driving two masses that are connected by a torsional spring as shown in the diagram below.



This system can represent a motor with a flexible shaft that drives a load. Assuming that the motor delivers a torque that is proportional to the current I, the dynamics of the system can be described by the equations,

$$J_1 \frac{d^2\phi_1}{dt^2} + c\left(\frac{d\phi_1}{dt} - \frac{d\phi_2}{dt}\right) + k(\phi_1 - \phi_2) = k_I I,$$

$$J_2 \frac{d^2\phi_2}{dt^2} + c\left(\frac{d\phi_2}{dt} - \frac{d\phi_1}{dt}\right) + k(\phi_2 - \phi_1) = T_d,$$

where ϕ_1 and ϕ_2 are the angles of the two masses, $\omega_i = d\phi_i/dt$ are their velocities, J_i represents moments of inertia, c is the damping coefficient, k represents the shaft stiffness, k_I is the torque constant for the motor, and T_d is the disturbance torque applied at the end of the shaft. Similar equations are obtained for a robot with flexible arms and for the arms of DVD and optical disk drives.

Derive a state space model for the system using the state variables $x_1 = \phi_1$, $x_2 = \phi_2$, $x_3 = \omega_1/\omega_0$, and $x_4 = \omega_2/\omega_0$, where $\omega_0 = \sqrt{\frac{k(J_1+J_2)}{J_1 J_2}}$.

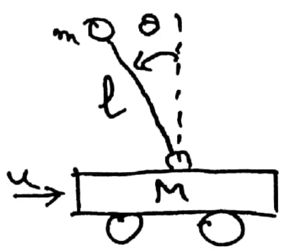
$\boxed{5 \text{ marks}}$

2. The dynamics of a simplified cart-pendulum system where the position of the base does not need to be controlled are,

$\boxed{5 \text{ marks}}$

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \frac{mgl}{J_t}\sin\theta - \frac{\gamma}{J_t}\dot{\theta} + \frac{l}{J_t}u\cos\theta \end{bmatrix}, \quad y = \theta$$

Find the equilibrium points. Discuss how these change as u increases from 0 to infinity.



3. A simplified model of bicycle dynamics are

$$J \frac{d^2\phi}{dt^2} - \frac{D v_0}{b}\frac{d\delta}{dt} = mgh\,\phi + \frac{m v_0^2 h}{b}\,\delta .$$

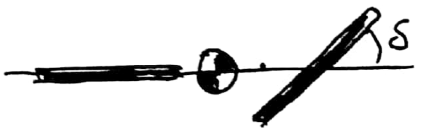
$\boxed{5 \text{ marks}}$

Show that these can be represented in state space form as

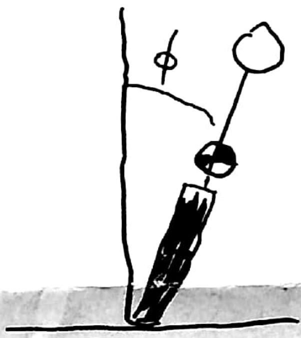
$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{mgh}{J} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} D v_0/bJ \\ m v_0^2 h/bJ \end{bmatrix} u,$$

$$y = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where input u is steering angle δ and output y is the tilt angle ϕ. If $u = 0$ what are the eigenvalues of the 'A' matrix of the above system?



TOP VIEW



REAR VIEW