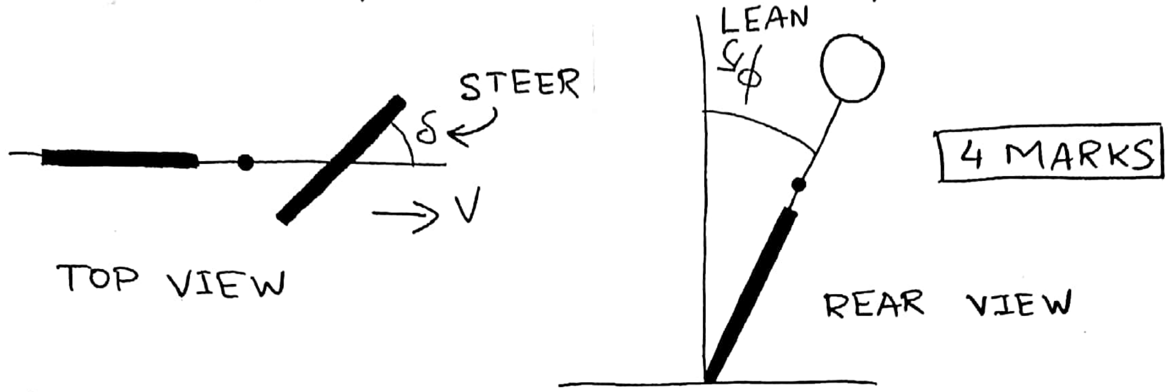
Prof Shaunak Sen

ELL 333 MINOR TEST 2 MARKS = 15 DURATION = 1 HOUR

1. The "inverted pendulum model" of a bicycle is,

$$J\frac{d^2\phi}{dt^2} - mgh\,\phi = \frac{DV}{b}\frac{d\delta}{dt} + \frac{mV^2h}{b}\,\delta,$$

where ϕ is the LEAN angle, δ is the STEER angle (and an input in this model), V is the forward velocity, and $\{J, m, g, h, D, b\}$ are bicycle parameters (all are positive).



4 MARKS

(a) Determine the stability of the upright position.

(b) A control strategy $\delta = -k\phi$ ("STEER in opposite direction to LEAN") is designed. Find the values of k for which the upright position is asymptotically stable.

(c) What happens to the minimum value of k for asymptotic stability when forward velocity increases?

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ENTRY NO.:=

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$\dot{z} = xy - \beta z$ NO input.

$\dot{y} = x(\rho - z) - y$ $\sigma, \rho, \beta \rightarrow$ parameters $\sigma>0, \rho>0, \beta>0$

$\dot{x} = \sigma(y - x)$ $x, y, z \rightarrow$ states

Q. The equations of motion of a "Lorenz System" are given below. This is a model of weather and may show chaos. What are the equilibrium points? Linearise the model about each of the equilibrium points.

ELL 333 QUIZ 1 MARKS = 7 DURATION = 20 MINUTES

2. Consider the linear system $\dot{x} = Ax$. Find $x(t)$ for

(a) $A = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}$, $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and

$\boxed{\text{4 MARKS}}$

(b) $A = \begin{bmatrix} 0 & -\omega & 1 & 0 \\ \omega & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix}$, $x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

DEF → ABC
BCD → AEF
ACE → BDF
ABF → CDE
ABC → DFE
BCD → EFA

using direct computation of matrix exponential or any other method.

3. Consider the system $\dot{x} = Ax$, $x \in \mathbb{R}^n$. Suppose A has no eigenvalues with a strictly positive real part and one or more eigenvalues with a zero real part. Explain why $x = 0$ is

- stable when the Jordan blocks corresponding to each eigenvalue with a zero real part are scalar (1X1) blocks.
- unstable when the Jordan block corresponding to at least one eigenvalue with a zero real part has size 2×2 or higher (it is not just a scalar 1×1 block).

$\boxed{\text{4 MARKS}}$

4. Consider the system $\dot{x} = A(t)x$, where

$\boxed{\text{3 MARKS}}$

$$A(t) = \begin{bmatrix} -1 + \frac{3}{2}\cos^2 t & 1 - \frac{3}{2}\cos t \cdot \sin t \\ -1 - \frac{3}{2}\cos t \cdot \sin t & -1 + \frac{3}{2}\sin^2 t \end{bmatrix}.$$

(a) Find the eigenvalues of $A(t)$.

(b) Show that $x(t) = \begin{bmatrix} -\epsilon e^{t/2}\cos t \\ \epsilon e^{t/2}\sin t \end{bmatrix}$, where $\epsilon > 0$ is a small positive number, is a solution of the system.

(c) Discuss the stability of $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ given the eigenvalues in part (a) and the solution in part (b).

Yash

THE END