ELL 409/791 Major Time: 100 mts Marks: 100

Q. 1. Write the locus of points on the decision boundary of a (i) n-input NOR gate and (ii) n-input NAND gate for variable number of inputs n.
(iii) Hence, determine for (i) and (ii), the weights and bias of a neuron whose output produces the corresponding functions.
(iv) Choose a specific map $\phi()$ from 2 dimensions to 3 dimensions to map samples of a 2 input XOR gate into a 3 dimensional space where the problem is linearly separable. Determine the locus of points in class 1 in the higher dimensional space.

(20)

Q. 2. A set of patterns $(x^i, y_i), x^i \in \Re^N, i = 1, 2, ..., M;\ y_i \in \{-1, 1\}$ is given. Formulate a linear programming problem that will allow you to determine when the dataset is not linearly separable.
(a) Write the objective function and constraints. Clearly explain how you will use the formulation to determine if a data set is linearly separable or not.
(b) If we are given a nonlinearly separable problem, how can we find which samples to remove so that the problem becomes linearly separable ?

(20)

Q. 3. A N bit A/D converter has N binary output variables $V_i, i = 0, 1, 2, ...N-1$. The input is an analog or continuous valued variable x. It is required to design a Hopfield net for the task of A/D conversion, i.e. to find values of $V_i \in \{0, 1\}$ so that the binary representation $V_{N-1}...V_0$ is as close as possible to the input value x.

1) Suggest an energy function whose minimum provides a solution to the problem of A/D conversion.
2) Determine the weights and offsets of the N neurons. Can you suggest a modification that allows the diagonal weights to be made zero ?

(20)

Q. 4. A set of patterns $x^k, k = 1, 2, ..., M$ is projected into a higher dimensional space by using a map $\phi()$. The inner products in the image space are given by the kernel function K.
(a) If the vector $\mu = \frac{1}{M}\sum \phi(x^i)$ denotes the mean of the image vectors, determine $||\mu||$ in terms of K.
(b) Determine $Max_i\ ||\phi(x^i)||$
(c) Determine $Min_i\ ||\phi(x^i)||$
Comment on the values of (b) and (c) for a RBF kernel.

(20)

Q. 5. A hyperplane classifier is given a set of samples $(x_1^i, x_2^i, x_3^i, x_4^i)^T$, $i = 1, 2, ..., 6000$ for training. (a) What is the probability that a learning algorithm will learn a set of weights so that the error rate on the general distribution exceeds 0.2 ?
(b) If the training error is ensured to be less than 10%, what is the probability that the test accuracy will be less than 80% ?

(20 marks)