20/11/'16 **ELL 700 Linear Systems** Time: 2 hrs.

MAJOR TEST Marks: 40

Q1: (a) Explain with necessary details why Tracking is not possible if plant T.F. $\hat{g}(s)$ has one or more zeros ~~at origin~~ at origin.

(b) Consider $\dot{x} = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$, $y = [2\ 0\ 0]x$

Let $u = pr - kx$. Find feedforward gain p and state feedback gain k so that resulting system has eigen values at -2 and $-1 \pm j1$ and will track asymptotically any step reference input.

(c) Show all the necessary modifications to part (b) including block diagrams and equations so that robust disturbance rejection & tracking is feasible. (15)

Q.2:- Given $\dot{x} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$, $y = [1\ 1]$

Find the state feedback gain k so that the state feedback system has -1 and -2 as its eigenvalues. Find k directly without using any equivalence transformation. (4)

Q.3:- Let $\dot{x} = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u$ with $x_1(0) = 10$ and $x_2(0) = -1$. ~~then~~

(a) Is it possible to transfer the initial state to origin in finite time. Give reasons. (4)

(b) If answer is yes in (a), then find the input which will transfer $X(0) = [10\ -1]^T$ to origin $[0\ 0]^T$ in 2 seconds.

Q.4:- Prove/Disprove that in LTV system if the pair $(A(t), B(t))$ is controllable at time t_0 then $W_c(t_0, t_1) = \int_{t_0}^{t_1} \Phi(t_1, \tau) B(\tau) B'(\tau) \Phi'(t_1, \tau)\, d\tau$ for some $t_1 > t_0$ (finite t_1) is non-singular (6)

Q.5:- Consider n dimensional, MIMO {p inputs, q outputs}

system $\dot{X} = AX + BU$, $Y = CX + DU$.

The rank of its controllability matrix is assumed to be $n_1 < n$. Let Q_1 be $n \times n_1$ matrix whose columns are any n_1 linearly independent columns of controllability matrix. Let P_1 be $n_1 \times n$ matrix such that $P_1 Q_1 = I_{n_1}$ (unit matrix of order n_1).

(a) Show whether following n_1 dimensional state eqn

$\dot{\bar{X}}_1 = P_1 A Q_1 \bar{X}_1 + P_1 B U$, $\bar{Y} = C Q_1 \bar{X}_1 + DU$ is controllable and has the same transfer matrix as original state eqn?

(b) The reduction procedure in part (a) requires solving for P_1 in $P_1 Q_1 = I_{n_1}$. Show how to solve for P_1. (7)

Q.6:- Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. Find A^{-1} using Cayley-Hamilton theorem. (4)